CURRENT WORKS OF KOREAN MATHEMATICIANS

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ABSTRACT. This paper is a collection of Abstracts of 17 articles consisting of an anthology on current studies of Korean mathematicians in the series entitled *Trends and Issues of Academic Research in Korea*, recently published by the National Academy of Sciences, Republic of Korea. The aim of the monograph is to introduce typical recent works of Korean mathematicians of high quality. Each author in the monograph is cited a large number of times by other scholars, according to MathSciNet.

1. Introduction

Since 2011 the National Academy of Sciences, Republic of Korea, has published a series of monographs entitled *Trends and Issues of Academic Research*, to provide scholars an in-depth perspective of current trends and controversial issues of academic research. The third monograph of the series, published in early 2014, is on the field of *Mathematics*, and the present author, in 2012, was assigned to be its editor.

One of the editor's tasks on the series is to write an introduction to the monograph other than routine editorial ones and the introduction should explain certain current trends of the assigned field for general readers.

The practical aim of the monograph on Mathematics is to introduce typical works of Korean mathematicians working all around the world in the past and present. Among the nearly one thousand such mathematicians, we chose seventeen authors who have many citations according to MathSciNet of the American Mathematical Society.

Each of the seventeen authors was asked to write a survey of his own major contributions, and to include some related unsolved problems, conjectures, and historical remarks. The detailed fields of mathematics that appear in the anthology include ring theory, associative algebra, representation theory, algebraic geometry, partial differential equations and applications, functional equations and inequalities, operator theory, nonlinear functional analysis, fixed point theory, algebraic topology, combinatorics, numerical analysis, optimization theory, operations research, and applied mathematics.

We hope that the anthology will help young Korean mathematicians learn what some of their senior colleagues have done, as well as to acquaint foreign scholars with some of the accomplishments of Korean mathematicians, by using the references to each of the articles in the anthology, along with English abstracts (using their 2010 MSC numbers and key words and phrases).

Our aim in the present paper is to introduce the contents of the anthology. We have collected abstracts of the seventeen papers in the anthology (in Section 2) and the list of their authors (in Section 3).

The Korean Mathematical Society has designated 2014 as the Year of Korean Mathematics, and is going to organize the International Congress of Mathematicians (ICM), which will meet in Seoul, Korea in August 2014. It is our wish that the monograph proves to be helpful to the participants in broadening their knowledge about Korean mathematics.

The monograph was distributed to major public libraries, university libraries, and mathematics departments in universities in Korea.

2. Abstracts of the seventeen articles

The third monograph of the series *Trends and Issues of Academic Research*, published by the NAS-ROK in 2014, is an anthology of seventeen articles concerning the current study of several detailed fields in Mathematics.

In this section we introduce the title, author, and abstract of each article in the anthology. Each article explains the major contribution of each author, with some related unsolved problems, conjectures and historical remarks.

We begin with an abstract of Introduction to the anthology, which is not contained in the seventeen articles.

Introduction to the anthology

Sehie Park

We introduce the overall areas of Mathematics and the detailed research fields in Mathematics and Mathematical Sciences according to 2010 Mathematics Subject Classification (2010 MSC) issued by the American Mathematical Society and the European Mathematical Society. We state general processes of mathematical research and underscore the importance of citations of research articles. We also recall the passage of twentieth century Mathematics into the one in the present century, the research situations of modern sciences in Korea, and some illustrations of progress of the Korean mathematical community. Finally, we state the editorial policy of this monograph.

The following are abstracts of the seventeen articles in the anthology, with 2010 MSC and Key words and phrases.

Rings of quotients and essential overrings

Jae Keol Park

This is mainly concerned with the study of right essential overrings of a ring and their properties. When a ring R is not right nonsingular, there appear numerous unexpected (also some pathological) results for right essential overrings. Thereby, the study for essential overrings is a new frontier topic in Ring Theory beyond maximal rings of quotients. We introduce various stimulating results and examples of right essential overrings which are not right rings of quotients of a ring R for further research.

The representation theory in Korea: Present and Future

Seok-Jin Kang

In this article, we give a brief account of main subjects in representation theory and discuss their developments. We also give a short description of the contributions made by Korean mathematicians such as Rim-Hak Ree, Bom-Shik Chang, Tae-il Suh, Byoung Song Chwe and Hyo Chul Myung. We then explain more recent developments made by Korean mathematicians in representation theory. The topics that we cover in this article include Kac-Moody algebras, quantum groups, crystal basis theory, Khovanov-Lauda-Rouquier algebras and higher representation theory.

Target rigidity of projective manifolds

Jun-Muk Hwang

Target rigidity arises in the study of deformations of surjective holomorphic maps between complex projective manifolds. A projective manifold X is target rigid if any surjective holomorphic map $f : Y \to X$ with X as a target cannot be deformed, with X and Y unchanged, in a nontrivial way. We survey known examples of projective manifolds with target rigidity with sketches of methods employed in the proof. Any projective manifold which is not uniruled is target rigid. Thus the major problem is when the manifold is uniruled. For uniruled projective manifolds, the method of varieties of minimal rational tangents introduced in the joint works of Ngaiming Mok and the author turns out to be an effective tool to study target rigidity. We review this method and exhibit how it can be applied to establish target rigidity of a large class of uniruled projective manifolds.

Inverse problems and experimental mathematics

Jin Keun Seo

Recently, imaging techniques in science, engineering, and medicine have evolved to expand our ability to visualize internal information of an object such as the human body. In particular, there has been marked progress in MR-based electromagnetic property imaging techniques which use MRI to provide cross-sectional images of conductivity, permittivity, and susceptibility distributions inside the human body. These electromagnetic material properties of biological tissues are important biomarkers since they reveal physiological and pathological conditions of body tissues and organs.

Since 2000, Seo and his group have focused mainly on mathematical theories with experimental validations such that the theories can guide experiments on what to seek. He collaborates with scientists in biomedical engineering and medical doctors, and his group has suggested various models including MREIT, the spectroscopic inverse admittivity problem (SIAP), a micro-EIT system, a transadmittance scanner, and others. This article focus on the multi-disciplinary area known as MREIT. After the invent of irrotational MREIT, imaging techniques in MREIT have been advanced rapidly and now can offer state-of-the-art conductivity imaging for animal and human experiments.

On the singularity problem of the Euler equations

Dongho Chae

The question of spontaneous apparition of singularity in the 3D incompressible Euler equations is one of the most important and challenging open problems in mathematical fluid mechanics. In this survey article we review some of recent approaches to the problem. We first review Kato's classical local well-posedness result in the Sobolev space and derive the celebrated Beale-Kato-Majda criterion for finite time blow-up. Then, we discuss recent refinements of the criterion as well as geometric type of theorems due to Constantin-Fefferman-Majda on the sufficiency condition for the regularity of solutions.

After that we review results excluding some of the scenarios leading to finite time singularities.

Functional equations and inequalities

Choonkil Park

In this note, we investigate functional equations and functional inequalities and we study their applications. First of all, we introduce the historic background for functional equations and their stability. We use the direct method and the fixed point method to prove the stability. Using the stability results of additive functional equations, we investigate morphisms in the following algebras: (1) C^*-algebra, (2) Lie C^*-algebra, (3) Jordan C^*-algebra, (4) Poisson JC^*-algebra, and (5) C^*-ternary algebra. We use the results on functional equations to investigate (1) fuzzy normed space, (2) random normed space, and (3) matrix normed space. Next, we study the extension process for the stability, and we investigate morphisms in the following vector spaces: (1) Banach module over C^*-algebra, (2) fuzzy normed space, (3) random normed space, (4) matrix normed space, and (5) matrix fuzzy normed space.

Bridge theory for Toeplitz operators

Woo Young Lee

Toeplitz operators arise naturally in several fields of mathematics and in a variety of problems in physics. Also the theory of hyponormal and subnormal operators is an extensive and highly developed area, which has made important contributions to a number of problems in functional analysis, operator theory, and mathematical physics. Thus, it becomes of central significance to describe in detail hyponormality and subnormality for Toeplitz operators. In this sense, the following question is challenging and interesting:

Which Toeplitz operators are hyponormal or subnormal?

While the precise relation between normality and subnormality has been extensively studied, as have been the classes of subnormal and hyponormal operators, the relative position of the class of subnormals inside the classes of hyponormals is still far from being well understood. We call it a "bridge theory" for operators to explore the gap between hyponormality and subnormality for bounded linear operators acting on an infinite dimensional complex Hilbert (or Banach) space. In this survey note, we provide a bridge theory for Toeplitz operators. This is originated from Halmos' Problem 5 (in 1970):

Is every subnormal Toeplitz operator either normal or analytic?

Even though Halmos' Problem 5 was, in 1984, answered in the negative by C. Cowen and J. Long, until now researchers have been unable to characterize subnormal Toeplitz operators in terms of their symbols. In this survey note we study the subnormality and hyponormality of Toeplitz operators acting on the vector-valued Hardy space $H^2_{\mathbf{C}^n}$ of the unit circle.

On the structure of Hilbert space operators:
some recent aspects

Il Bong Jung

The invariant subspace problem is contained in the center to study the structure of Hilbert space operators, which was suggested by John von Neumann in 1930's. We introduce some history and its background about invariant subspace problem in this article. Also we give some recent aspects and related results. And we glimpse the characterizations for operators between subnormal and hyponormal operators. Such study is called the gap theory of operators. We overview several related problems and their aspects in the gap theory. Striving operator gaps, we usually use and apply bounded classical weighted shifts. We introduce weighted shifts on directed trees which generalize the classical weighted shifts. Finally, we discuss some operator properties about them and further related studies.

Iterative methods for nonlinear problems and applications

Yeol Je Cho

In this survey, based on Picard's convergence theorem and Banach's fixed point theorem, we introduce the following: (1) some kinds of iterative methods, (2) Halpern's theorem and some related results with open problems, (3) generalized equilibrium problems and related applications, (4) hierarchical fixed point and variational inequality problems and related results, (5) multiple–sets split feasibility problems and related results, and finally, (6) bilebel mixed equilibrium problems and related results.

Strong convergences and weak convergences based on diverse iterative schemes

Byung-Soo Lee

In this paper, we consider the strong convergences and the weak convergences of sequences $\{x_n\}$ generated by the diverse iterative schemes (Mann iterative scheme, Halpern iterative scheme, Ishikawa iterative scheme, Noor iterative scheme and multi-step iterative scheme) to some fixed point (resp., common fixed point) of various kinds of mappings (resp., a family of mappings) under suitable conditions.

Iterative methods for zeros of accretive operators

Jong Soo Jung

We introduce several iterative methods for finding zeros of accretive operators in Banach spaces and provide several convergence theorems for approximating fixed points of nonexpansive mappings and pseudocontractive mappings. We present convergence theorems of several iterative methods for nonexpansive mappings and pseudocontractive mappings.

The fixed point method versus the KKM method

Sehie Park

The partial KKM principle for an abstract convex space is an abstract form of the classical Knaster-Kuratowski-Mazurkiewitz theorem. A partial KKM space is an abstract convex space satisfying the partial KKM principle. We show that how the fundamental theorems on equilibrium problems

can be extended to abstract convex spaces. Precisely, most of important results in the KKM theory hold without assuming the linearity in topological vector spaces. Moreover, we compare the fixed point method and the KKM method in nonlinear analysis. By applying such two methods, we obtained very general forms of the following important theorems: (1) the von Neumann minimax theorem, (2) the von Neumann intersection lemma, (3) the Nash equilibrium theorem, (4) the social equilibrium existence theorem of Debreu, (5) the Gale-Nikaido-Debreu theorem, (6) the Fan-Browder fixed point theorem, (7) generalized Fan minimax inequality, (8) an existence theorem for solutions of generalized quasi-equilibrium problems, (9) the Himmelberg fixed point theorem, and (10) a general KKM type theorem.

On the Hurwitz covering enumeration problem

Jin Ho Kwak

At the end of the nineteenth century, Hurwitz constructed a generating function for the number of nonequivalent coverings of the Riemann sphere having only simple branch points (of order two). His two papers became a source of the so-called Hurwitz Enumeration Problem: *Determine the number of branched coverings of a given Riemann surface with prescribed ramification type.* Some partial solutions of the problem has been obtained, but still is a famous open problem.

The Hurwitz numbers are important in topology as an enumeration problem, but also important on their connections with the geometry of the moduli space of curves, mathematical physics, and group theory.

This paper is devoted to solutions of the enumeration problem for (branched) coverings of Riemann surfaces and, more generally of graphs and manifolds with finitely generated fundamental groups. We represent here some well-known results in this field and recent developments of the problem, and indicate a general approach to solve the problem in high-dimensional case. In this survey we cover group theoretical, combinatorial and topological points of view on the problem.

symmetry, group representation, character, Möbius function, Euler function, von Sterneck-Ramanujan function, Burnside counting lemma.

Real hypersurfaces in Hermitian symmetric spaces with rank 2

YOUNG JIN SUH

The purpose of this article is to introduce some recent developments for real hypersurfaces in Hermitian symmetric spaces with rank 2. There are many examples in a class of Hermitian symmetric spaces with rank 2. But among them we focus on complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2}) = SU(m+2)/S(U(2){\cdot}U(m))$, complex hyperbolic two-plane Grassmannians $G_2^*(\mathbb{C}^{m+2}) = SU(2,m)/S(U(2){\cdot}U(m))$ and complex quadrics $\mathbb{Q}^m = SO(m+2)/S(2)SO(m)$.

Moreover, we have tried to give some detailed explanations about historical backgrounds and some important theorems for real hypersurfaces in above Hermitian symmetric spaces of rank 1 or rank 2 established until now.

Floer homology in symplectic topology and mirror symmetry

YONG-GEUN OH

In this survey, the author reviews what the Floer homology is and what it does in symplectic geometry both in the closed string and in the open string context. In the first case, the author will explain how the chain level Floer theory leads to the C^0 symplectic invariants of Hamiltonian flows and to the study of topological Hamiltonian dynamics. In the second case, the author explains how Floer's original construction of Lagrangian intersection Floer homology is obstructed as soon as one leaves the category of exact Lagrangian submanifolds. Some applications of this general machinery to the study of topology of Lagrangian embeddings in relation to symplectic topology and to the mirror symmetry are also reviewed.

This article is mainly a survey article slightly modified and updated version of our previous survey article published in the Madrid ICM-2006 Proceedings.

Some remarkable results in numerical analysis
and applied mathematics

Dongwoo Sheen

We summarize several results concerning Green's theorem dealing with function spaces arising from electro-magnetics. The traces of tangential fields of H(curl) space is analyzed in the sense of Sobolev spaces of negative index. Several cutting-edge nonconforming finite element spaces have been introduced. The Laplace transform method for solving evolution problems is introduced. The method is embarrassingly parallel and converges exponentially.

2010 Mathematics Subject Classification: 35A22, 35K10, 35Q61, 65N30, 64R10, 65Y05, 76M10.

Key words and phrases: Green's theorem, trace theorem, nonconforming finite element, parallel method, Laplace transform.

Optimality conditions and stability analysis
for optimization problems

Gue Myung Lee

We give a sequential optimality condition for a convex optimization problem which holds without any constraint qualifications and an optimality condition which holds under a weakened constraint qualification.

We summarized relations among the solution sets of the vector convex optimization problem involving non-differential convex functions and the solution sets of vector variational inequalities expressed with subdifferentials, necessary and sufficient conditions to be the solutions of vector variational inequalities when their constraint sets are given by convex functions, and the result of stability analysis of perturbed strongly monotone vector variational inequalities and its application to perturbed strongly convex vector optimization problems.

We present the characterization of stable Farkas lemma when the objective function of a convex optimization problem is perturbed by linear functionals and give the stable Lagrange duality theorem for the problem as its application.

Moreover, we give the results of the stability analysis for perturbed quadratic optimization problems, which are about the continuity of Karush-Kuhn-Tucker point multifunction, the continuity of the optimal solution multifunction and the calculation formula for the directional derivative and the derivative of the optimal value function.

2010 Mathematics Subject Classification: 49J52, 49J53, 49K40, 90C20, 90C25, 90C26, 90C29, 90C30, 90C46.

Key words and phrases: optimization problem, convex function, constraint qualification, optimality condition, vector variational inequality, vector convex optimization problem, solution set, perturbed strongly monotone vector variational inequality, perturbed strongly convex vector optimization problem, stable Farkas lemma, stable Lagrange duality theorem, quadratic optimization problem, stability analysis, sensitivity analysis, KKT-point multifunction, optimal solution multifunction, optimal value function.

3. The seventeen authors: Affiliations, E-mail addresses and Major fields

The following is the list of the seventeen authors of articles in the anthology with their affiliations, e-mail addresses, and the fields given mainly in MathSciNet for which their major works were done.

In the following list, the order is given alphabetically by the family name of each author.

Chae, Dongho, Distinguished Professor, Chung-Ang University, Seoul
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PDE and Flued mechanics

Cho, Yeol Je, Professor, Kyungsang National University, Jinju
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Nonassociative rings and algebras, Representation theory

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Algebraic topology and Combinatorics

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Difference and Functional equations

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Associative rings and algebras

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