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Recent results in analytical fixed point theory

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Abstract

We survey recent results in analytical fixed point theory. Firstly, we state resolutions of long-standing problems in infinite dimensional topology—the Schauder conjecture, the compact AR problem, and the Banach problem on the Hilbert cube. Secondly, we list some fixed point theorems on Kakutani maps, generalized upper hemicontinuous maps, Fan–Browder maps, approximable maps, acyclic maps, and admissible or better admissible class $\mathfrak{B}$ of maps. Some related results are also given.

Keywords: Schauder conjecture; AR problem; Banach problem; Kakutani map; Admissible set; Better admissible class of multimaps

In this survey, we review some fixed point theorems on multimaps (maps) and related results mainly due to the present author. The terminology and notations are standard. For overall historical background, see [11].

1. The Schauder conjecture and other problems

For a long period it was not known whether it was true in every topological vector space (t.v.s.) that every compact convex subset has the fixed point property (the Schauder conjecture), whether every compact convex subset of a metrizable t.v.s. is an AR (the

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compact AR problem), and whether every infinite dimensional compact convex subset of a metrizable t.v.s. is homeomorphic to the Hilbert cube (the Banach problem).

In 2001, Robert Cauty [4] obtained the affirmative solution to the Schauder conjecture as follows:

Theorem 1 (*Cauty [4]*). *Let E be a Hausdorff t.v.s., C a convex subset of E, and f a continuous function from C into C. If $f(C)$ is contained in a compact subset of C, then f has a fixed point.*

Historically well-known partial forms of Theorem 1 were obtained by Brouwer (1912), Schauder (1927, 1930), Tychonoff (1935), Hukuhara (1950), Klee (1960), Fan (1964), Idzik (1987), Nhu (1996), Arandelović (1996), and others. For the literature, see [11,14,16].

A *polytope* P in a t.v.s. E is a nonempty compact convex subset of E contained in a finite dimensional subspace of E. A nonempty subset X of a t.v.s. E is said to be *admissible* (in the sense of Klee) provided that, for every compact subset K of X and every neighborhood V of the origin 0 of E, there exists a continuous function $h : K \to X$ such that $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a polytope in X.

Note that every nonempty convex subset of a locally convex Hausdorff t.v.s. is admissible. Other examples of admissible t.v.s. are l^p, L^p, the Hardy spaces H^p for $0 < p < 1$, the space $S(0, 1)$ of equivalence classes of measurable functions on $[0, 1]$, and others. Moreover, any locally convex subset of an F-normable t.v.s. and any compact convex locally convex subset of a t.v.s. is admissible. Note that an example of a nonadmissible nonconvex compact subset of the Hilbert space l^2 is known.

From now on, all metrizable t.v.s. is assumed to be separable. Motivated by Dobrowolski [5], we obtained the following in [17]:

Lemma 1. *A compact convex subset of a metrizable t.v.s. is admissible.*

From Lemma 1, in 2003, the present author [17] obtained affirmative solutions to the compact AR problem and the Banach problem as follows:

Theorem 2. *A compact convex subset of a metrizable t.v.s. is an AR.*

Theorem 3. *An infinite dimensional compact convex subset of a metrizable t.v.s. is homeomorphic to the Hilbert cube Q.*

Significant consequences of Theorems 2 and 3 are that all *Roberts spaces*—that is, compact convex sets with no extreme points constructed by Roberts' method [21] of needle point spaces—are AR and have the fixed point property; see [17].

2. Kakutani maps

From now on, we are mainly concerned with multimaps (or simply, maps) $T : X \multimap Y$ for subsets X and Y of t.v.s. A *Kakutani map* is an upper semicontinuous (u.s.c.) multimap with nonempty closed convex values. The following are recently obtained:

Theorem 4 (*Dobrowolski [6]*). *Let X be a compact convex subset of a Hausdorff t.v.s. Then every Kakutani map* $T : X \multimap X$ *has a fixed point.*

Theorem 5 (*Dobrowolski [6]*). *Let X be a convex subset of a metrizable t.v.s. Then every compact Kakutani map* $T : X \multimap X$ *has a fixed point.*

Particular forms of Theorem 4 are due to Kakutani (1941), Bohnenblust and Karlin (1950), Fan (1952), Glicksberg (1952), Granas and Liu (1986), Park (1988), Okon (2002), and others. See [7–12,16].

These theorems can be immediately applied to the Leray–Schauder type alternatives. We give only one as follows:

Theorem 6. *Let X be a compact convex subset of a Hausdorff t.v.s. and U an open subset of X such that* $0 \in U$. *Let* $T : \overline{U} \multimap X$ *be a Kakutani map. Then one of the following holds*:

(i) *T has a fixed point.*
(ii) *There exists an* $x \in \mathrm{Bd}\, U$ *and a* $\lambda \in (0, 1)$ *such that x is a fixed point of* λT.

For a metrizable t.v.s., the compactness of *X* in Theorem 6 can be replaced by the compactness of *T* by Theorem 5.

For Kakutani maps, we have another fixed point theorem.

Theorem 7 (*Park and Tan [20]*). *Let X be a subset of a locally convex Hausdorff t.v.s. E and Y an almost convex dense subset of X. Let* $T : X \multimap X$ *be a compact u.s.c. map with nonempty closed values such that* $T(y)$ *is convex for all* $y \in Y$. *Then T has a fixed point.*

In particular, for $Y = X$, we obtain

Corollary 7.1. *Let X be an almost convex subset of a locally convex Hausdorff t.v.s. Then any compact Kakutani map* $T : X \multimap X$ *has a fixed point in X.*

Moreover we have the following almost fixed point theorem [15]:

Theorem 8. *Let X be a subset of a t.v.s. and Y an almost convex dense subset of X. Let* $T : X \multimap E$ *be a l.s.c.* [*u.s.c.*] *map such that* $T(y)$ *is convex for all* $y \in Y$. *If there is a totally bounded subset K of* $\overline{X}$ *such that* $T(y) \cap K \neq \emptyset$ *for each* $y \in Y$, *then for any convex neighborhood U of the origin 0 of E, there exists a point* $x_U \in Y$ *such that* $T(x_U) \cap (x_U + U) \neq \emptyset$.

3. Generalized upper hemicontinuous maps

The upper semicontinuity is generalized to upper demicontinuity (u.d.c.), to upper hemicontinuity (u.h.c.), and to generalized upper hemicontinuity. The Kakutani fixed point theorem has a large number of generalizations.

A *convex space* X is a nonempty convex set with any topology that induces the Euclidean topology on the convex hulls of its finite subsets. A nonempty subset L of a convex space X is called a *c-compact set* if for each finite set $N \subset X$ there is a compact convex set $L_N \subset X$ such that $L \cup N \subset L_N$. Let $[x, L]$ denote the closed convex hull of $\{x\} \cup L$ in X, where $x \in X$.

Let $cc(E)$ denote the set of nonempty closed convex subsets of a t.v.s. E and $kc(E)$ the set of nonempty compact convex subsets of E. Let E^* denote the topological dual of E.

Let $X \subset E$ and $x \in E$. The *inward* and *outward sets* of X at x, $I_X(x)$ and $O_X(x)$, are defined as follows:

$$I_X(x) = x + \bigcup_{r>0} r(X - x), \quad O_X(x) = x + \bigcup_{r<0} r(X - x).$$

For $p \in \{\mathrm{Re}\, h : h \in E^*\}$ and $U, V \subset E$, let

$$d_p(U, V) = \inf\{|p(u - v)| : u \in U, \ v \in V\}.$$

The following is the basis of the main theorem in [8,14]:

Theorem 9. *Let X be a convex space, L a c-compact subset of X, K a nonempty compact subset of X, E a t.v.s. containing X as a subset, and $F : X \to 2^E \setminus \{\emptyset\}$. Suppose that, for each $p \in \{\mathrm{Re}\, h : h \in E^*\}$,*

(0) *$p|_X$ is continuous on X;*
(1) *$X_p = \{x \in X : \sup p(Fx) \geqslant p(x)\}$ is compactly closed in X;*
(2) *$x \in K$ and $p(x) = \max p(X)$ implies $x \in X_p$; and*
(3) *$x \in X \setminus K$ and $p(x) = \max p[x, L]$ implies $x \in X_p$.*

Then there exists an $x \in \bigcap\{X_p : p \in \{\mathrm{Re}\, h : h \in E^*\}\}$.

Remarks. 1. In Theorem 9, we do not require any concrete connection between topologies of X and E except (0). This is why there have appeared fixed point theorems on maps whose domains and ranges have different topologies.

2. If F is u.h.c. on each nonempty compact subset C of X, then F satisfies the "continuity" condition (1) for all $p \in \{\mathrm{Re}\, h : h \in E^*\}$, but not conversely. Any map F satisfying (1) is said to be *generalized u.h.c.*

3. The "boundary" condition (2) is equivalent to the following:

(2)′ $x \in K$ and $p(x) = \max p(\overline{I_X(x)})$ implies $x \in X_p$.

In fact, $p(x) = \max p(X)$ is equivalent to $p(x) = \max p(\overline{I_X(x)})$.

Let X be a nonempty convex subset of a vector space E. The *algebraic boundary* $\delta_E(X)$ of X in E is the set of all $x \in X$ for which there exists $y \in E$ such that $x + ry \notin X$ for all $r > 0$. If E is a t.v.s., the *topological boundary* $\mathrm{Bd}\, X = \mathrm{Bd}_E X$ of X is the complement of $\mathrm{Int}_E X$ in $\overline{X}$. It is known that $\delta_E(X) \subset \mathrm{Bd}\ X$ and in general $\delta_E(X) \neq \mathrm{Bd}\ X$.

Moreover, the "boundary" condition (2)′ is equivalent to the following:

(2)″ $x \in K \cap \delta_E(X)$ and $p(x) = \max p(\overline{I_X(x)})$ implies $x \in X_p$.

4. The "coercivity" or "compactness" condition (3) is equivalent to the following:

(3)′ $x \in X \setminus K$ and $p(x) = \max p(\overline{I_L(x)})$ implies $x \in X_p$.

Theorem 10. *Let X be a convex space, L a c-compact subset of X, K a nonempty compact subset of X, E a Hausdorff t.v.s. containing X as a subset, and F a map satisfying either*

(A) *E^* separates points of E and $F : X \to kc(E)$, or*
(B) *E is locally convex and $F : X \to cc(E)$.*

(I) *Suppose that for each $p \in \{\mathrm{Re}\, h : h \in E^*\}$,*

(0) *$p|_X$ is continuous on X;*
(1) *$X_p = \{x \in X : \inf p(Fx) \leqslant p(x)\}$ is compactly closed in X;*
(2) *$d_p(Fx, \overline{I_X(x)}) = 0$ for every $x \in K \cap \delta_E(X)$; and*
(3) *$d_p(Fx, \overline{I_L(x)}) = 0$ for every $x \in X \backslash K$.*

Then there exists an $x \in X$ such that $x \in F(x)$.

(II) *Suppose that for each $p \in \{\mathrm{Re}\, h : h \in E^*\}$,*

(0) *$p|_X$ is continuous on X;*
(1)′ *$X_p = \{x \in X : \sup p(Fx) \geqslant p(x)\}$ is compactly closed in X;*
(2)′ *$d_p(Fx, \overline{O_X(x)}) = 0$ for every $x \in K \cap \delta_E(X)$; and*
(3)′ *$d_p(Fx, \overline{O_L(x)}) = 0$ for every $x \in X \backslash K$.*

Then there exists an $x \in X$ such that $x \in F(x)$. Further, if F is u.h.c., then $F(X) \supset X$.

Recall that Theorem 10 unifies, improves and generalizes historically well-known fixed point theorems published in nearly 50 papers; see the diagram in [8,11,14].

4. The Fan–Browder maps

Usually, a multimap with nonempty convex values and open fibers is called a *Browder map*. The well-known *Fan–Browder fixed point theorem* states that a Browder map from a compact convex subset of a t.v.s. into itself has a fixed point [3].

In 1992, Ben-El-Mechaiekh conjectured that the Fan–Browder theorem might be true if we assume that the Browder map is compact instead of the compactness of its domain. In 2003, we showed that this conjecture is affirmative as follows [18]:

Theorem 11. *Let X be a convex subset of a Hausdorff t.v.s. and $F : X \multimap X$ a compact Browder map. Then F has a fixed point.*

This can be applied to economic equilibrium problems and others.

Earlier, we obtained a number of generalizations of the Fan–Browder fixed point theorem; see [13]. The following is one of them:

Theorem 12. *Let X be a convex space and $F : X \multimap X$ a multimap with open* [*resp. closed*] *fibers. If X is covered by a finite number of fibers of F, then* co *F has a fixed point.*

For a topological space X and a convex space Y, a multimap $T : X \multimap Y$ is called a *Φ-map* or a *Fan–Browder map* provided that there exists a multimap $S : X \multimap Y$ such that

(1) for each $x \in X$, $\operatorname{co} S(x) \subset T(x)$; and
(2) $X = \bigcup\{\operatorname{Int} S^{-}(y) : y \in Y\}$, where $S^{-}(y) = \{x \in X : y \in S(x)\}$.

Corollary 12.1. *Let X be a convex space, K a nonempty compact subset of X, and $S, T : X \multimap X$ multimaps. Suppose that*

(1) *for each $x \in X$, $\operatorname{co} S(x) \subset T(x)$;*
(2) $K \subset \bigcup\{\operatorname{Int} S^{-}(z) : z \in X\}$; *and*
(3) *for each finite subset N of X, there exists a compact convex subset L_N of X containing N such that*

$$L_N \backslash K \subset \bigcup\{\operatorname{Int} S^{-}(z) : z \in L_N\}.$$

Then T has a fixed point.

More general versions of Theorems 11 or 12 were established for more general classes of Fan–Browder maps and locally selectionable maps with convex values; see [19].

5. Approximable maps

Given two open neighborhoods U and V of the origin 0 of t.v.s. E and F, respectively, and for $X \subset E$ and $Y \subset F$, a (U, V)-*approximative continuous selection* of $T : X \multimap Y$ is a continuous function $s : X \to Y$ satisfying

$$s(x) \in (T[(x + U) \cap X] + V) \cap Y \quad \text{for } x \in X.$$

A multimap $T : X \multimap Y$ is said to be *approachable* if it admits a (U, V)-approximative continuous selection for every U and V as above.

A multimap $T : X \multimap Y$ is said to be *approximable* if its restriction $T|_K$ to any compact subset K of X is approachable.

Note that an approachable map is always approximable. Recall that Ben-El-Mechaiekh and Deguire [1] and Ben-El-Mechaiekh and Idzik [2] established a large number of properties and examples of approachable or approximable maps.

Recently, we obtained the following results in [19]:

Theorem 13. *Let X be a convex subset of a Hausdorff t.v.s. E and $T : X \multimap X$ a compact closed approachable map. Then T has a fixed point.*

Theorem 14. *Let X be a closed subset of a Hausdorff t.v.s. E such that* $0 \in \operatorname{Int} X$ *and* $T : X \multimap E$ *a compact closed approachable map. Then either*

(1) *T has a fixed point or*
(2) $\lambda x \in T(x)$ *for some* $\lambda > 1$ *and* $x \in \operatorname{Bd} X$.

From the above Leray–Schauder alternative, as in [10], we can deduce a Schaefer-type theorem, Birkhoff–Kellogg-type theorems on eigenvalues or invariant directions, fixed point theorems for non-self maps, and quasi-variational inequalities, all related to compact closed approachable maps; see [19].

For compact approximable maps, we have the following:

Theorem 15. *Let X be a compact convex subset of a Hausdorff t.v.s. E. Then every closed approximable map* $T : X \multimap X$ *has a fixed point.*

Theorem 16. *Let X be an admissible convex subset of a Hausdorff t.v.s. and* $T : X \multimap X$ *a compact closed approximable map. Then T has a fixed point.*

Theorem 17. *Let X be a closed subset of a Hausdorff t.v.s. E such that* $0 \in \mathrm{Int}\, X$, *Y an admissible convex subset of E containing X, and* $T : X \multimap Y$ *a compact closed approximable map. Then either*

(1) *T has a fixed point or*
(2) $\lambda x \in T(x)$ *for some* $\lambda > 1$ *and* $x \in \mathrm{Bd}_Y X$.

From this result, as for Theorem 14, we can deduce various consequences all related to compact closed approximable maps.

6. Acyclic maps

A topological space is said to be *acyclic* if all of its reduced Čech homology groups over the rational field vanish. A u.s.c. map is said to be *acyclic* if it has compact acyclic values.

The following is known in 1991 [7]:

Theorem 18. *Let X and C be nonempty convex subsets of a locally convex Hausdorff t.v.s. E. Let* $T : X \multimap X + C$ *be a compact acyclic map. Suppose that one of the following holds*:

(1) *X is closed and C is compact.*
(2) *X is compact and C is closed.*
(3) $C = \{0\}$.

Then there is an $x_0 \in X$ *such that* $T(x_0) \cap (x_0 + C) \neq \emptyset$.

For a convex-valued map *T*, Theorem 18 is due to Lassonde (1983). Later in 1997, Theorem 18 was extended to the better admissible class of maps [12].

The following case (3) of Theorem 18 is known to be very useful in many applications:

Corollary 18.1. *Let X be a convex subset of a locally convex Hausdorff t.v.s. Then every compact acyclic map* $T : X \multimap X$ *has a fixed point.*

Corollary 18.1 was extended to the class $\mathfrak{A}_c$ (1993), to $\mathbb{V}_c^\sigma$ (1994), to $\mathfrak{A}_c^\sigma$ (1994), and to $\mathfrak{B}$ (1997). Later, this is extended to a non-locally convex t.v.s.; see Theorems 19 and 20 below.

7. Admissible classes of multimaps

Let X and Y be topological spaces. An *admissible class* $\mathfrak{A}_c^\kappa(X, Y)$ of maps $T : X \multimap Y$ is one such that, for each compact subset K of X, there exists a map $S \in \mathfrak{A}_c(K, Y)$ satisfying $S(x) \subset T(x)$ for all $x \in K$; where $\mathfrak{A}_c$ consists of finite compositions of maps in $\mathfrak{A}$, and $\mathfrak{A}$ is a class of maps satisfying the following properties:

(1) $\mathfrak{A}$ contains the class $\mathbb{C}$ of (single-valued) continuous functions;
(2) each $F \in \mathfrak{A}_c$ is u.s.c. and compact-valued; and
(3) for each polytope P, each $T \in \mathfrak{A}_c(P, P)$ has a fixed point, where the intermediate spaces of compositions are suitably chosen for each $\mathfrak{A}$.

Examples of $\mathfrak{A}$ are continuous functions $\mathbb{C}$, the Kakutani maps $\mathbb{K}$, the Aronszajn maps $\mathbb{M}$ (with R_δ values), the acyclic maps $\mathbb{V}$, the Powers maps $\mathbb{V}_c$, the O'Neil maps $\mathbb{N}$ (continuous with values of one or m acyclic components, where m is fixed), the approachable maps $\mathbb{A}$ (whose domains and codomains are subsets of uniform spaces), admissible maps of Górniewicz, σ-selectional maps of Haddad and Lasry, permissible maps of Dzedzej, and others. Further, the Fan–Browder maps (codomains are convex sets), locally selectionable maps having convex values, $\mathbb{K}_c^\sigma$ due to Lassonde, $\mathbb{V}_c^\sigma$ due to Park et al., and approximable maps $\mathbb{A}_c^\kappa$ due to Ben-El-Mechaiekh and Idzik are examples of $\mathfrak{A}_c^\kappa$. For the literature, see [9,12,11,15].

For a subset X of a t.v.s. E, we defined the "*better*" admissible class $\mathfrak{B}$ of maps as follows [12]:

$T \in \mathfrak{B}(X, X) \iff T : X \multimap X$ is a map such that for any polytope P in X and any continuous function $f : T(P) \to P$, the composition $f(T|_P) : P \multimap P$ has a fixed point.

Note that $\mathfrak{A}_c^\kappa(X, X) \subset \mathfrak{B}(X, X)$ for a convex subset X of a t.v.s. E, and some examples of maps in $\mathfrak{B}$ not belonging to $\mathfrak{A}_c^\kappa$ are known. Moreover, compact closed maps in the class KKM due to Chang and Yen and in the class s-KKM due to Chang et al. also belong to the class $\mathfrak{B}$.

The following fixed point theorems were obtained in the late 1990s by the author [12,11]:

Theorem 19. *Let E be a Hausdorff t.v.s. and X an admissible compact convex subset of E. Then any map $T \in \mathfrak{A}_c^\kappa(X, X)$ has a fixed point.*

Theorem 20. *Let E be a Hausdorff t.v.s. and X an admissible convex subset of E. Then any compact closed map $T \in \mathfrak{B}(X, X)$ has a fixed point.*

In [12], it was shown that Theorem 20 subsumes more than 60 known or possible particular cases and generalizes them in terms of the involving spaces and multimaps as well.

It is not known whether the admissibility of X can be eliminated in Theorems 19 and 20. If so, these results would be far-reaching generalizations of the Cauty Theorem 1.

In view of Lemma 1, we obtain the following new results:

Corollary 19.1. *Let K be a compact convex subset of a metrizable t.v.s. E. Then any* $T \in \mathfrak{A}_c^{\kappa}(K, K)$ *has a fixed point.*

Corollary 20.1. *Let K be a compact convex subset of a metrizable t.v.s. E. Then any closed* $T \in \mathfrak{B}(K, K)$ *has a fixed point.*

8. Further results

There have appeared a number of papers by other authors concerned with generalizations, variations, or applications of our works.

The concept of compact maps has variants (not necessarily generalizations) in that of various types of condensing maps (pseudo-condensing or countably condensing maps or of Mönch type). It is well-known that the theory of such types of condensing maps reduces to that of compact maps. Therefore, our theorems might be applied to those types of condensing maps.

In our previous works, as applications of some of our fixed point theorems, we obtained results on best approximations, variational or quasi-variational inequalities, minimax inequalities, saddle points in nonconvex sets, openness of multimaps, existence of maximal elements, the Walras excess demand theorem, acyclic versions of the Nash equilibrium theorem, generalized equilibrium problems, quasi-equilibrium problems, generalized complementarity problems, and others.

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