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Leray-Schauder Alternatives for Approximable Maps in Topological Vector Spaces

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Abstract—Using a fixed-point theorem for compact approximable maps defined on an admisssible convex subset of a topological vector space, we prove Leray-Schauder alternatives for compact or pseudo-condensing approximable maps.

Keywords—Leray-Schauder alternative, Approximable map, Topological vector space, Admissible set.

1. INTRODUCTION

In [1], some fixed-point theorems of Leray-Schauder type for compact single-valued functions in topological vector spaces were obtained. In [2], certain related results for compact multimaps in locally convex topological vector spaces were given. Further, some Leray-Schauder type theorems for compact or condensing approximable maps in locally convex topological vector spaces were proved in [3–5].

The aim in this paper is to establish Leray-Schauder alternatives for approximable maps in topological vector spaces. The concept of admissibility due to Klee [6] plays an important role in the fixed-point theory in topological vector spaces. It is well known that every convex subset of a locally convex topological vector space is admissible.

We first give a fixed-point theorem for compact approximable maps defined on an admisssible convex subset of a topological vector space and then obtain a Leray-Schauder alternative for compact approximable maps in topological vector spaces. The main result includes many known results, see, for example, [1–3,5].

Next, using the notion of c-measure of noncompactness introduced in [7], we obtain a Leray-Schauder type theorem for pseudocondensing approximable maps in topological vector spaces.

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Moreover, we obtain a Leray-Schauder alternative for quasi-compact or ultimately compact maps in topological vector spaces. Note that there have appeared interesting studies on essential maps of approximable and acyclic type, see [8].

In this paper, all topological vector spaces are assumed to be Hausdorff. For a subset K of a topological vector space E, its closure, convex hull, and closed convex hull in E are denoted by $\bar{K}$, $\operatorname{co} K$, and $\overline{\operatorname{co}}\, K$, respectively. For a subset U of K, its boundary and interior with respect to the relative topology on K are denoted by $\partial_K U$ and $\operatorname{Int}_K U$, respectively.

A *multimap* or *map* $T : X \multimap Y$ is a function from a set X into the set of all nonempty subsets of a set Y. For topological spaces X and Y, a map $T : X \multimap Y$ is said to be *upper semicontinuous* on X if for any open set V in Y, the set $\{x \in X : T(x) \subset V\}$ is open in X. T is said to be *closed* if it has a closed graph; and *compact* if its range $T(X)$ is contained in a compact subset of Y.

Let $T : X \multimap Y$ be a map where X and Y are subsets of topological vector spaces E and F, respectively. Given two open neighborhoods U and V of the origins in E and F, respectively, a continuous function $s : X \to Y$ is a *(U, V)-approximative continuous selection* of T if $s(x) \in (T[(x + U) \cap X] + V) \cap Y$ for each $x \in X$.

T is said to be *approachable* if it has a (U, V)-approximative continuous selection for every open neighborhoods U and V of the origins in E and F, respectively. T is said to be *approximable* if its restriction $T|_K$ to any compact subset K of X is approachable. $\mathcal{A}(X, Y)$ denotes the class of all approximable maps $T : X \multimap Y$. For details, we refer to [3,9].

A nonempty subset X of a topological vector space E is said to be *admissible* provided that, for every compact subset K of X and every neighborhood V of the origin 0 in E, there exists a continuous function $h : K \to X$ such that $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a finite-dimensional subspace L of E. If $X = E$, then the topological vector space E is said to be *admissible*. See [6,10].

Recall that every nonempty convex subset of a locally convex topological vector space is admissible; see [11,12]. The spaces $L^p(0, 1)$ for $0 < p < 1$ and $S(0, 1)$ are admissible topological vector spaces; see [13,14].

2. COMPACT APPROXIMABLE MAPS

Applying the fixed-point property of admissible convex sets in a topological vector space, we prove a Leray-Schauder alternative for compact approximable maps in topological vector spaces.

We begin with the following lemma.

LEMMA 2.1. *Let X be an admissible convex set in a topological vector space E and $T : X \multimap X$ a compact closed map such that $T \in \mathcal{A}(X, \overline{T(X)})$. Then T has a fixed point.*

PROOF. Let $\mathcal{N}$ be a fundamental system of neighborhoods of the origin 0 of E and $V \in \mathcal{N}$. Since $C := \overline{T(X)}$ is a compact subset of the admissible set X, there exist a continuous function $h : C \to X$ and a finite-dimensional subspace L of E such that $y - h(y) \in V$ for all $y \in C$ and $h(C) \subset L$.

Let $M := h(C)$ and $K := \operatorname{co} M$. Since M is a compact subset of $L \cap X$, it follows that K is a compact convex subset of $L \cap X$. Since $T|_K : K \multimap C$ admits a (V, V)-approximative continuous selection, there exists a continuous function $s : K \to C$ such that $s(x) \in (T[(x + V) \cap K] + V) \cap C$ for all $x \in K$.

Consider the composition $h \circ s : K \to K$ given by $h \circ s(x) = h(s(x))$ for $x \in K$. The Brouwer fixed-point theorem implies that there exists an $x_V \in K$ such that $h \circ s(x_V) = x_V$. Setting $y_V = s(x_V) \in C$, we have $y_V - h(s(x_V)) = y_V - h(y_V) \in V$ and $s(x_V) - w_V \in V$ for some $w_V \in T(z_V)$ such that $z_V \in (x_V + V) \cap K$. By the compactness of K in X, we may suppose that $h(s(x_V)) = x_V \to x$ for some $x \in K$. Hence, $z_V \to x$ and $s(x_V) = y_V \to x$ and so $w_V \to x$. Since T is closed and $w_V \in T(z_V)$, we have $x \in T(x)$. This completes the proof. ∎

For the fixed-point theory of more general better admissible maps, see [15].

Corollary 2.2. *(See [1, Bemerkung 4].) Let X be an admissible convex set in a topological vector space E. If $f : X \to X$ is a compact continuous function, then f has a fixed point.*

Now we can prove the main theorem in this section.

Theorem 2.3. *Let X be a closed convex admissible subset of a topological vector space E, $u \in K$, and $K \subset X$ a closed neighborhood of u in X. Let $T : K \multimap X$ be a compact closed map such that $T \in \mathcal{A}(K, \overline{T(K)})$. Then the following Leray-Schauder alternative holds:*

(1) *T has a fixed point; or*
(2) *there exist a point $x \in \partial_X K$ and a number $\lambda > 1$ such that $\lambda x + (1-\lambda)u \in T(x)$.*

Proof. Assume that $x \notin T(x)$ for all $x \in K$ and $T(y) \cap \{\lambda y + (1-\lambda)u : \lambda > 1\} = \emptyset$ for all $y \in \partial_X K$. Without loss of generality, we may suppose that a fundamental basis $\mathcal{N}$ of E consists of circled open neighborhoods of 0 in E. Consider

$$R := \{x \in K : x \in tT(x) + (1-t)u, \text{ for some } t \in [0,1]\}.$$

Then it is clear that R is closed, and hence, compact in X because T is a compact closed map and K is closed in X. Thus, $R \cap \partial_X K = \emptyset$. Since X as a subset of Hausdorff topological vector space is completely regular, there is a continuous function $h : X \to [0,1]$ such that $h(x) = 1$ for every $x \in R$ and $h(x) = 0$ for every $x \in \partial_X K$. Let a map $S : X \multimap X$ be defined by

$$S(x) := \begin{cases} h(x)T(x) + (1-h(x))u, & \text{for } x \in K, \\ \{u\}, & \text{for } x \in X \setminus \mathrm{Int}_X K. \end{cases}$$

Then S is a compact closed map, since T is a compact closed map, h is continuous, and X is a convex subset of E with $u \in X$. Moreover, $S : X \multimap \overline{S(X)}$ is approximable. By definition of S, it suffices to show that for any compact set C in K, $S|_C : C \multimap \overline{S(X)}$ is approachable. For every $(U, V) \in \mathcal{N} \times \mathcal{N}$, if $s : C \to \overline{T(K)}$ is a (U, V)-approximative continuous selection of $T|_C$, then

$$h(x)s(x)+(1-h(x))u \in h(x)(T[(x+U)\cap C]+V)+(1-h(x))u \subset S[(x+U)\cap C]+V, \quad \text{for each } x \in C.$$

For each $x \in C$, we have

$$h(x)s(x) + (1-h(x))u \in \overline{\{h(x)y + (1-h(x))u : x \in K, y \in T(x)\}} = \overline{S(X)}.$$

Thus, $S|_C : C \multimap \overline{S(X)}$ is approachable.

By Lemma 2.1, S has a fixed point $x_0 \in X$; that is, $x_0 \in S(x_0)$. If $x_0 \notin K$, then $S(x_0) = \{u\}$ and so $u = x_0 \notin K$, which contradicts $u \in K$. Hence, $x_0 \in K$. Now $x_0 \in S(x_0) = h(x_0)T(x_0) + (1-h(x_0))u$ implies $x_0 \in R$ and so $h(x_0) = 1$. Therefore, we have $x_0 \in T(x_0)$ which contradicts our assumption. This completes the proof. ∎

Corollary 2.4. *(See [3, Theorem].) Let K be a closed subset of a locally convex topological vector space E such that $0 \in \mathrm{Int}_E K$. Suppose that $T : K \multimap E$ is a compact upper semicontinuous map with closed values such that $T \in \mathcal{A}(K, \overline{T(K)})$. If T is fixed-point-free, then there exists a point $(\lambda, x) \in (0,1) \times \partial_E K$ such that $x \in \lambda T(x)$.*

Corollary 2.5. *(See [1, Hauptsatz 2].) Let X be a closed convex admissible subset of a topological vector space E with $0 \in X$, and $K \subset E$ a closed neighborhood of 0 in E. Let $f : K \cap X \to X$ be a compact continuous function such that for each $x \in X \cap \partial_E K$, $f(x) = \alpha x$ implies $\alpha \leq 1$. Then f has a fixed point.*

Remark. Theorem 2.3 remains true even if the condition "X is closed" is dropped.

3. PSEUDOCONDENSING APPROXIMABLE MAPS.

Using the notion of c-measure of noncompactness [7], we obtain a Leray-Schauder type theorem for pseudocondensing approximable maps in topological vector spaces.

Let E be a topological vector space, K a nonempty subset of E, A a cone in a vector space with the partial ordering $\leq$, and γ a collection of subsets of $\overline{\text{co}}\, K$ with the property that for any $M \in \gamma$, the sets $\bar{M}, \text{co}\, M, M \cup \{u\}$ $(u \in K)$ and every subset of M belong to γ. Let c be a real number with $c \geq 1$. A function $\Phi : \gamma \to A$ is called *a c-measure of noncompactness* on K provided that the following conditions hold for any $M \in \gamma$:

(1) $\Phi(\bar{M}) = \Phi(M)$;
(2) if $x \in K$, then $\Phi(M \cup \{x\}) = \Phi(M)$;
(3) if $N \subset M$, then $\Phi(N) \leq \Phi(M)$;
(4) $\Phi(\text{co}\, M) \leq c\Phi(M)$.

Let X and K be nonempty subsets of E such that $X \subset K$ and let Φ be a c-measure of noncompactness on K. A map $T : X \multimap K$ is said to be *pseudocondensing* provided that if B is any subset of X such that $\Phi(B) \leq c\Phi(T(B))$, then B is relatively compact in X. In particular, if $c = 1$, then T is called *condensing*. See [7,16].

A subset K of a topological vector space E is said to be *locally convex* if for every $x \in K$ there exists a base of neighborhoods $U(x)$ of x in K such that $U(x) = W(x) \cap K$ and $W(x)$ is a convex subset of E.

Any subset of a locally convex topological vector space is a locally convex set. Every closed, convex, and locally convex set in a topological vector space is admissible; see [7].

The following property of pseudocondensing maps is a basic tool for the main result of this section.

LEMMA 3.1. *Let X and K be nonempty subsets of a topological vector space E such that $X \subset K$. If $T : X \multimap K$ is a pseudocondensing map, there exists a nonempty closed convex subset B of E such that $B \cap X$ is a compact subset of X and $T(B \cap X) \subset B$.*

PROOF. Let $x_0 \in X$ be given and let

$$\Sigma := \{C \subset E : C = \overline{\text{co}}\, C,\ x_0 \in C,\ T(C \cap X) \subset C\}.$$

Then Σ is nonempty because $\overline{\text{co}}\,(T(X) \cup \{x_0\}) \in \Sigma$. Let $B := \bigcap_{C \in \Sigma} C$ and $B_1 := \overline{\text{co}}\,(T(B \cap X) \cup \{x_0\})$. Since B is closed and convex, $x_0 \in B$, and $T(B \cap X) \subset C$ for all $C \in \Sigma$, we have $B \in \Sigma$. Hence, $B_1 \subset B$ and so $T(B_1 \cap X) \subset T(B \cap X) \subset B_1$. Therefore $B_1 \in \Sigma$. It follows by definition of B that $B \subset B_1$, and hence, $B = \overline{\text{co}}\,(T(B \cap X) \cup \{x_0\})$. Since Φ is a c-measure of noncompactness, we have $\Phi(B \cap X) \leq c\Phi(T(B \cap X) \cup \{x_0\}) = c\Phi(T(B \cap X))$. Since T is pseudocondensing and B is closed in E, we conclude that $B \cap X$ is a compact subset of X. This completes the proof. ∎

We now obtain the following result on pseudocondensing approximable maps in topological vector spaces. A similar result for condensing maps in locally convex topological vector spaces is given in [4, Theorem].

THEOREM 3.2. *Let K be a closed convex subset of a topological vector space E, $u \in X$, and $X \subset K$ a closed neighborhood of u in K. Suppose that every closed convex subset of E is admissible. If $T : X \multimap K$ is a pseudocondensing upper semicontinuous map with compact values and $T \in \mathcal{A}(X, \overline{T(X)})$, then one of the following properties holds:*

(1) *T has a fixed point; or*
(2) *there exist a point $x \in \partial_K X$ and a number $\lambda \in (0,1)$ such that $x \in \lambda T(x) + (1-\lambda)u$.*

PROOF. Assume that $x \notin T(x)$ for all $x \in X$ and $y \notin \lambda T(y) + (1-\lambda)u$ for all $(\lambda, y) \in (0,1) \times \partial_K X$. Since $T : X \multimap K$ is pseudocondensing, by Lemma 3.1, there exists a nonempty closed convex subset B of E such that $B \cap X$ is a compact subset of X and $T(B \cap X) \subset B$. We may suppose that $u \in B$. Since $T \in \mathcal{A}(X, \overline{T(X)})$, $T|_{B\cap X} : B \cap X \multimap \overline{T(X)}$ is approachable.

Consider the set $C := \{x \in B \cap X : x \in \lambda T(x) + (1 - \lambda)u, \text{ for some } \lambda \in [0,1]\}$. Since T is an upper semicontinuous map with compact values and $B \cap X$ is compact in X, T is closed, and the set $T(B \cap X)$ is compact in K. Hence, C is a nonempty compact subset of K. Since $C \cap \partial_K X = \emptyset$, there is a continuous function $h : K \to [0,1]$ such that $h(x) = 1$ for every $x \in C$ and $h(x) = 0$ for every $x \in \partial_K X$.

Let $S : B \multimap B$ be a map defined by

$$S(x) := \begin{cases} h(x)T(x) + (1 - h(x))u, & \text{for } x \in B \cap X, \\ \{u\}, & \text{for } x \in B \setminus X. \end{cases}$$

Since $T(B \cap X)$ is compact in K and T has closed graph, S is a compact closed map. As in the proof of Theorem 2.3, since $T|_{B\cap X} : B \cap X \multimap \overline{T(X)}$ is approachable, it is easy to check that $S|_{B\cap X} : B \cap X \multimap \overline{S(B)}$ is approachable, and hence, $S : B \multimap \overline{S(B)}$ is approximable.

Since the closed convex subset B of E is admissible, by Lemma 2.1, S has a fixed point $x_0 \in B$; that is, $x_0 \in S(x_0)$. Clearly, $x_0 \in B \cap X$. Now $x_0 \in S(x_0) = h(x_0)T(x_0) + (1 - h(x_0))u$ implies that $x_0 \in C$, and hence, $h(x_0) = 1$. Thus, x_0 is a fixed point of T, contrary to our assumption. This completes the proof. ∎

COROLLARY 3.3. *Let X be a closed neighborhood of 0 in a locally convex topological vector space E. Suppose that $T : X \multimap E$ is a condensing upper semicontinuous map with compact values such that $T \in \mathcal{A}(X, \overline{T(X)})$. If T is fixed-point-free, then there exists a point $(\lambda, x) \in (0,1) \times \partial_E X$ such that $x \in \lambda T(x)$.*

4. QUASI-COMPACT AND ULTIMATELY COMPACT MAPS

In this section, we present a Leray-Schauder alternative for quasi-compact or ultimately compact maps in topological vector spaces.

Let U and X be nonempty subsets of a topological vector space E. A map $T : U \multimap X$ is said to be *quasi-compact* if there exists a closed convex subset S of E with the property that $U \cap S \neq \emptyset$, $T(U \cap S) \subset S$, and $\overline{T(U \cap S)}$ is compact in X. Such a set S is called *a characteristic set* for T. See [17,18].

For a map $T : U \multimap E$, let $R_0 := \overline{\text{co}}\, T(U)$ and $R_\alpha := \overline{\text{co}}\, T(U \cap R_{\alpha-1})$ if α is an ordinal of the first kind, and $R_\alpha := \bigcap_{\beta<\alpha} R_\beta$ if α is an ordinal of the second kind. The set R_δ for an ordinal δ satisfying:

$$R_\alpha = R_\delta, \qquad \text{for all } \alpha \geq \delta$$

is denoted by R^*. T is said to be *ultimately compact* if $\overline{T(U \cap R^*)}$ is compact in E. See [18–20].

LEMMA 4.1. *Let X be a closed, convex and locally convex set in a topological vector space E and $T : X \multimap X$ a quasi-compact upper semicontinuous map with closed values such that $T|_M \in \mathcal{A}(M, \overline{T(M)})$ for every closed, convex and locally convex subset M of E with $M \subset X$. Then T has a fixed point.*

PROOF. Choose a closed convex subset S of E such that $X \cap S \neq \emptyset$, $T(X \cap S) \subset S$ and the set $\overline{T(X \cap S)}$ is compact in X. Hence, the set $M := X \cap S$ is a closed, convex and locally convex set in E and so is admissible; and the restriction $T_0 := T|_M : M \multimap M$ is a compact closed map and $T_0 \in \mathcal{A}(M, \overline{T_0(M)})$. By Lemma 2.1, T_0 has a fixed point. This completes the proof. ∎

Finally, we show that the following Leray-Schauder alternative for quasi-compact maps reduces to that for ultimately compact maps.

THEOREM 4.2. *Let X be a closed neighborhood of $u \in X$ in a topological vector space E such that every closed convex subset of E is admissible. Let $T : X \multimap E$ be a quasi-compact closed* *map with a characteristic set S containing u such that $T|_M \in \mathcal{A}(M, \overline{T(M)})$ for every closed subset M of X. Then the following Leray-Schauder alternative holds:*

(1) *T has a fixed point; or*

(2) *there exist a point $x \in \partial_S(X \cap S)$ and a number $\lambda > 1$ such that $\lambda x + (1-\lambda)u \in T(x)$.*

PROOF. Since $T : X \multimap E$ is a quasi-compact map, there exists a closed convex subset S of E with $T(X \cap S) \subset S$ such that $T(X \cap S)$ is relatively compact in E. Set $K := X \cap S$ and let $T_0 := T|_K : K \multimap S$ be the restriction of T to K. Then K is a closed neighborhood of u in S, and S is admissible. Moreover, it is clear that T_0 is a compact closed map and $T_0 \in \mathcal{A}(K, \overline{T_0(K)})$. By Theorem 2.3, T_0 has a fixed point or there exist a point $x \in \partial_S K$ and a number $\lambda > 1$ such that $\lambda x + (1-\lambda)u \in T_0(x)$. This completes the proof. ∎

COROLLARY 4.3. *Let X be a closed neighborhood of 0 in a locally convex topological vector space E. Suppose that $T : X \multimap E$ is an ultimately compact closed map such that R^* contains 0 and $T|_M \in \mathcal{A}(M, \overline{T(M)})$ for every closed subset M of X. If T is fixed-point-free, then there exist a point $x \in \partial_{R^*}(X \cap R^*)$ and a number $\lambda > 1$ such that $\lambda x \in T(x)$.*

PROOF. Since $T : X \multimap E$ is ultimately compact, the set $\overline{T(X \cap R^*)}$ is compact in E. From $R^* = \overline{\text{co}}\, T(X \cap R^*)$ it follows that $T(X \cap R^*) \subset R^*$. Hence, T is a quasi-compact map with characteristic set R^*. In view of Theorem 4.2, the desired conclusion follows from the fact that every closed convex subset of locally convex topological vector space E is admissible. This completes the proof. ∎

COROLLARY 4.4. *Let X be a closed neighborhood of 0 in a locally convex topological vector space E and $f : X \to E$ be a quasi-compact continuous function with a characteristic set S containing 0. If for each $x \in S \cap \partial_E X$, $f(x) = \alpha x$ implies $\alpha \le 1$, then f has a fixed point.*

PROOF. We can apply Corollary 2.5 because the restriction $f|_{X \cap S} : X \cap S \to S$ is a compact continuous function and S is a closed convex admissible set in E. This completes the proof. ∎

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