REMARKS ON FIXED POINT THEOREMS FOR GENERALIZED CONVEX SPACES

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Abstract. If $(X, D; \Gamma)$ is a Φ-space, then any closed compact map $F \in \mathfrak{B}(X, X)$ has a fixed point. This extends recent theorems due to Ben-El-Mechaiekh *et al.* [B], Yuan [Y], and Wu [W]. Moreover, we apply our results to the Kakutani maps and answer a question of Kirk and Shin [KS] affirmatively.

Dedicated to Dr. Hyun-Chun Shin who founded Gyeongsang National University as the first President.

0. Introduction

Motivated by the well-known works of Horvath [H1,2] on C-spaces, the author introduced the concept of generalized convex spaces or G-convex spaces which have been extensively studied by Park *et al.* [PK2-5, P5,7-9].

At the International Conference, Chinju, Korea, Aug. 4-5, 1998, the author introduced the contents of Kim [K] and Park [P7,8]. At that time or a little earlier at the NACA'98, Niigata, Japan, July 28-31, 1998, the author became aware of works of Ben-El-Mechaiekh *et al.* [B] and Yuan [Y], and then immediately found that their works are particular to the main result of [P8].

Our aim in this paper is to clarify this fact as a continuation of [P8]. Note that our argument seems to be much simpler than that of [B], [Y]. Moreover, we apply our results to the Kakutani type multimaps for C-spaces and obtain an affirmative answer to a problem raised by Kirk and Shin [KS] for hyperconvex metric spaces.

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In this paper, all topological spaces are assumed to be Hausdorff and t.v.s. means topological vector spaces. For notations and terminology, we follow [P3-6].

1. Better admissible maps on G-convex spaces

A *generalized convex space* or a *G-convex space* $(X, D; \Gamma)$ consists of a topological space X, a nonempty subset D of X, and a multimap $\Gamma : \langle D \rangle \multimap X$ such that for each $A \in \langle D \rangle$ with the cardinality $|A| = n+1$, there exists a continuous function $\phi_A : \Delta_n \to \Gamma(A)$ such that $J \in \langle A \rangle$ implies $\phi_A(\Delta_J) \subset \Gamma(J)$. Note that $\phi_A|_{\Delta_J}$ can be regarded as ϕ_J.

Here, $\langle D \rangle$ denotes the set of all nonempty finite subsets of D, Δ_n the standard n-simplex, and Δ_J the face of Δ_n corresponding to $J \in \langle A \rangle$. We may write $\Gamma_A = \Gamma(A)$ for each $A \in \langle D \rangle$ and $(X, \Gamma) = (X, X; \Gamma)$. A subset C of X is said to be Γ-*convex* if for each $A \in \langle D \rangle$, $A \subset C$ implies $\Gamma_A \subset C$. For details on G-convex spaces, see [PK2-5, P5,7-9], where basic theory was extensively developed.

There are a lot of new examples of G-convex spaces; see [P9].

Let $(X, D; \Gamma)$ be a G-convex space and Y a topological space. We define *the better admissible class* $\mathfrak{B}$ of multimaps from X into Y as follows:

$F \in \mathfrak{B}(X, Y) \iff F : X \multimap Y$ is a multimap such that for any $N \in \langle D \rangle$ with $|N| = n+1$ and any continuous map $p : F(\Gamma_N) \to \Delta_n$, the composition

$$\Delta_n \xrightarrow{\phi_N} \Gamma_N \overset{F|_{\Gamma_N}}{\multimap} F(\Gamma_N) \xrightarrow{p} \Delta_n$$

has a fixed point.

For convex spaces, the class $\mathfrak{B}$ appears in [P3-6].

For any topological space E and a G-convex space $(X, D; \Gamma)$, a map $T : E \multimap X$ is called a Φ-*map* if there exists a map $S : E \multimap D$ such that

(i) for each $y \in E$, $M \in \langle Sy \rangle$ implies $\Gamma_M \subset Ty$; and
(ii) $E = \bigcup \{\operatorname{Int} S^- x : x \in D\}$.

The concept of Φ-maps is originated from Horvath [H1] and motivated by the works of Fan and Browder; see [P2].

For a particular type of G-convex spaces, we can establish fixed point theorems for the class $\mathfrak{B}$ as follows:

A G-convex space $(X, D; \Gamma)$ is a Φ-*space* if X is a separated uniform space and for each entourage V there is a Φ-map $T : X \multimap X$ such that $\mathrm{Gr}(T) \subset V$. This concept is originated from Horvath [H1], where a number of examples were given.

The following is our main result of [P8] whose proof is given here for the completeness:

Theorem 1. *Let $(X, D; \Gamma)$ be a Φ-space and $F \in \mathfrak{B}(X, X)$. If F is closed and compact, then F has a fixed point.*

Proof. Let $\mathcal{V} = \{V_\lambda\}_{\lambda \in I}$ be a basis of the separated uniform structure of X. Let $K = \overline{F(X)}$ be the closure of the range of F. Since $(X, D; \Gamma)$ is a Φ-space, for each $\lambda \in I$, there is a Φ-map $T_\lambda : X \multimap X$ such that $\mathrm{Gr}(T_\lambda) \subset V_\lambda$. Since K is compact, it is known that $T_\lambda|_K$ has a continuous selection $f_\lambda : K \to \Gamma_N$ for some $N \in \langle D \rangle$ such that $f_\lambda = \phi_N \circ p$, where $p : K \to \Delta_n$ is a continuous map; see [P8]. Since $F \in \mathfrak{B}(X, K)$, the composition

$$\Delta_n \xrightarrow{\phi_N} \Gamma_N \overset{F|_{\Gamma_N}}{\multimap} F(\Gamma_N) \subset K \xrightarrow{p} \Delta_n$$

has a fixed point $a_\lambda \in \Delta_n$; that is, $a_\lambda \in (p \circ F \circ \phi_N) a_\lambda$. Hence,

$$x_\lambda := \phi_N(a_\lambda) \in (\phi_N \circ p \circ F) x_\lambda = (f_\lambda \circ F) x_\lambda$$

and there exists a $y_\lambda \in F(x_\lambda) \subset K$ such that $x_\lambda = f_\lambda(y_\lambda) \in T_\lambda(y_\lambda)$; that is, $(x_\lambda, y_\lambda) \in V_\lambda$. Therefore

$$(x_\lambda, y_\lambda) \in V_\lambda \cap \mathrm{Gr}(F) \subset X \times K.$$

Since K is compact, we may assume that $\{y_\lambda\}_{\lambda \in I}$ converges to some $x_0 \in K$. Since $(x_\lambda, y_\lambda) \in V_\lambda$ for all $\lambda \in I$, $\{x_\lambda\}_{\lambda \in I}$ also converges to $x_0 \in K$. Since F is closed and $(x_\lambda, y_\lambda) \in \mathrm{Gr}(F)$, we should have $(x_0, x_0) \in \mathrm{Gr}(F)$. Therefore, F has a fixed point $x_0 \in K$.

Particular forms of Theorem 1 were known by Horvath [H1] and Park and Kim [PK1].

2. Approachable maps on G-convex spaces

Recently, Ben-El-Mechaiekh *et al.* [B] introduced the class $\mathbb{A}$ of approachable multimaps as follows:

Let X and Y be topological spaces. A multimap $F : X \multimap Y$ is said to be *approachable* whenever

(i) X and Y are uniformizable (with respective bases $\mathcal{U}$ and $\mathcal{V}$ of symmetric entourages), and

(ii) F admits a continuous W-approximative selection $s : X \to Y$ for each W in the basis $\mathcal{W}$ of the product uniformity on $X \times Y$; that is, $\mathrm{Gr}(s) \subset W[\mathrm{Gr}(F)]$, where

$$W[A] = \bigcup_{z \in A} W[z] = \{z' \in X \times Y : W[z'] \cap A \neq \emptyset\}$$

for any $A \subset X \times Y$, and

$$W[z] = \{z' \in X \times Y : (z, z') \in W]$$

for $z \in X \times Y$.

We denote $F \in \mathbb{A}(X, Y)$ if $F : X \multimap Y$ is approachable.

The following two lemmas are [B, Lemmas 2.4 and 4.1], respectively.

Lemma 1. *Let $(X, \mathcal{U})$, $(Y, \mathcal{V})$, $(Z, \mathcal{W})$ be three uniform spaces, with Z compact, and let $\Psi : Z \multimap X$, $\Phi : X \multimap Y$ be two upper semicontinuous(u.s.c.), closed-valued approachable maps. Then so is their composition $\Phi \circ \Psi$.*

Lemma 2. *If X is a nonempty convex subset of a locally convex t.v.s. and if $\Phi \in \mathbb{A}(X, X)$ is u.s.c. with closed values, then Φ has a fixed point provided that Φ is compact.*

From Lemmas 1 and 2, we show that certain approachable maps are better admissible if their domains are G-convex spaces as follows:

Lemma 3. *Let* $(X, D; \Gamma)$ *be a* G*-convex uniform space with the basis* $\mathcal{U}$ *and* $(Y, \mathcal{V})$ *a uniform space. If* $F \in \mathbb{A}(X, Y)$ *is closed compact, then* $F \in \mathfrak{B}(X, Y)$.

Proof. For any $N \in \langle D \rangle$ with $|N| = n + 1$ and continuous map $p : F(\Gamma_N) \to \Delta_n$, consider the composition

$$\Delta_n \xrightarrow{\phi_N} \Gamma_N \overset{F|_{\Gamma_N}}{\multimap} F(\Gamma_N) \xrightarrow{p} \Delta_n.$$

Since F is closed and compact, F is u.s.c. with closed values, and hence so is $F|_{\phi_N(\Gamma_N)}$. Note also that each of ϕ_N, $F|_{\Gamma_N}$ and p is approachable. Therefore, by Lemma 1, the composition $p \circ (F|_{\Gamma_N}) \circ \phi_N$ is an upper semicontinuous approachable map with nonempty closed values. Now, by Lemma 2, it has a fixed point. This completes our proof.

From Theorem 1 and Lemma 3, we have

Theorem 2. *Let* $(X, D; \Gamma)$ *be a* Φ*-space and* $F \in \mathbb{A}(X, X)$. *If* F *is closed and compact, then* F *has a fixed point.*

This extends the main theorem of Ben-El-Mechaiekh *et al.* [B, Theorem 4.2], which is the case $X = D$ under the additional restriction that $Sy \subset Ty$ for each $y \in E$ in the definition of Φ-maps. Similarly, other results of Ben-El-Mechaiekh *et al.* [B, Corollaries 4.3, 4.4, 4.7, and 4.8] can also be improved.

3. Acyclic maps on G-convex spaces

The study of fixed points of compact acyclic maps in a t.v.s. was initiated by the author in 1991; see [P1]. For the subsequent works of the author; see [P2,4,6].

For topological spaces X and Y, we adopt the following:

$F \in \mathbb{V}(X, Y) \iff F : X \multimap Y$ is an acyclic map; that is, an u.s.c. multimap with compact acyclic values.

$F \in \mathbb{V}_c(X,Y) \Longleftrightarrow F : X \multimap Y$ is a finite composition of acyclic maps where the intermediate spaces are topological.

It is known that $\mathbb{V}_c(X,Y) \subset \mathfrak{B}(X,Y)$ whenever X is a G-convex space, and that any map in $\mathbb{V}_c$ is closed.

The following was introduced in [P7]:

A *locally G-convex uniform space* is a G-convex space $(X, D; \Gamma)$ such that

(1) X is a separated uniform space with the basis $\mathcal{V}$;

(2) D is dense in X; and

(3) for each $V \in \mathcal{V}$ and each $x \in X$,

$$V[x] = \{x' \in X : (x, x') \in V\}$$

is Γ-convex.

Particular types of locally G-convex uniform spaces are treated recently by Yuan [Y] for the case $X = D$ and by Wu [W] for the case $X = D$ is a C-space.

Lemma 4. *A locally G-convex uniform space $(X, D; \Gamma)$ is a Φ-space.*

Proof. Let $\mathcal{V}$ be an (open) basis of the uniform structure of $(X, D; \Gamma)$. For each $V \in \mathcal{V}$, define $T : X \multimap X$ and $S : X \multimap D$ by

$$Tx := \{y \in X : (x, y) \in V\}$$

and

$$Sx := \{y \in D : (x, y) \in V\}$$

for $x \in X$. Since D is dense in X, for each $x \in X$, there is a $y \in D$ such that $(x, y) \in V$; hence

$$x \in S^{-}y = T^{-}y.$$

Since $T^{-}y$ is open, we have $X = \bigcup\{\operatorname{Int} S^{-}y : y \in D\}$. Moreover, for each $x \in X$, if $M \in \langle Sx \rangle \subset \langle D \rangle$, then

$$\Gamma_M \subset \{y \in X : (x, y) \in V\} = Tx.$$

Therefore S and T satisfy conditions (i) and (ii) of the definition of a Φ-map.

From Lemma 4 and Theorem 1, we have the following:

Theorem 3. *Let* $(X, D; \Gamma)$ *be a locally* G*-convex uniform space. Then any closed compact map* $F \in \mathfrak{B}(X, X)$ *has a fixed point.*

Since $\mathbb{V}_c(X, X) \subset \mathfrak{B}(X, X)$, we have the following:

Corollary. *Let* $(X, D; \Gamma)$ *be a locally* G*-convex uniform space. Then any compact map* $F \in \mathbb{V}_c(X, X)$ *has a fixed point.*

Note that Yuan [Y, Theorem 2.1] obtained a particular form of Corollary for the case $X = D$ itself is compact and for $\mathbb{V}$ instead of $\mathbb{V}_c$. Similarly, other results in [Y] also can be improved.

Note also that Wu [W, Theorem 1] is a particular form of [Y, Theorem 2.1] for the case $X = D$ is a C-space.

4. The Kakutani maps on G-spaces

Usually, an u.s.c. multimap with nonempty (closed) convex values is called a Kakutani map within the frame of t.v.s. In this section, we obtain fixed point theorems for general Kakutani type multimaps defined on particular types of G-spaces.

We give a well-known subclass of G-convex spaces due to Horvath [H2] as follows:

A pair (X, Γ) is called a *C-space* (or an *H-space*) if X is a topological space and $\Gamma : \langle X \rangle \multimap X$ a multimap such that $\Gamma_A \subset \Gamma_B$ for $A \subset B$ in $\langle X \rangle$ and each Γ_A is ω-connected (that is, n-connected for all $n \geq 0$). We can also define a locally C-convex uniform space as in Section 3.

A C-space (X, Γ) is called an *LC-space* if X is a separated uniform space and if there exists a basis $\mathcal{V}$ for the symmetric entourages such that for each $V \in \mathcal{V}$, $\{x \in X : C \cap V[x] \neq \emptyset\}$ is Γ-convex whenever $C \subset X$ is Γ-convex; see [H2].

Lemma 5. *Every LC-space* (X, Γ) *is locally C-convex if every singleton is* Γ*-convex.*

Proof. For each entourage V and any $x \in X$,

$$\begin{aligned} V[x] &= \{x' \in X : (x, x') \in V\} \\ &= \{x' \in X : x \in V[x']\} \\ &= \{x' \in X : \{x\} \cap V[x'] \neq \emptyset\}. \end{aligned}$$

Since $\{x\}$ is Γ-convex and (X, Γ) is an LC-space, $V[x]$ is Γ-convex. Therefore, (X, Γ) is locally C-convex.

The following is due to Ben-El-Mechaiekh *et al.* [B, Proposition 3.9]:

Lemma 6. *Let* $(X, \mathcal{U})$ *and* $(Y, \mathcal{V})$ *are uniform spaces. If either*

(i) X *is paracompact and* (Y, Γ) *is an LC-space; or*

(ii) X *is compact and* (Y, Γ) *is a G-convex space,*

then every u.s.c. map $F : X \multimap Y$ *with nonempty* Γ*-convex values is approachable; that is,* $F \in \mathbb{A}(X, Y)$.

From Theorem 2 and Lemma 6(ii), we have

Theorem 4. *Let* (X, Γ) *be a compact* Φ*-space. Then any u.s.c. map* $F : X \multimap X$ *with nonempty closed* Γ*-convex values has a fixed point.*

From Theorem 3 and Lemmas 5 and 6(i), we have the following due to Ben-El-Mechaiekh *et al.* [B, Corollary 4.7]:

Theorem 5. *Let* (X, Γ) *be a paracompact LC-space such that every singleton is* Γ*-convex. Then any compact u.s.c. map with nonempty closed* Γ*-convex values has a fixed point.*

5. The Kakutani maps on hyperconvex spaces

The notion of hyperconvex spaces was introduced by Aronszajn and Panitchpakdi [AP] in 1956. In 1979, independently Sine [Si] and Soardi [So] proved the fixed point property for nonexpansive maps on bounded hyperconvex spaces. Since then many interesting works have appeared for hyperconvex spaces; for the literature, see [K, P10–12].

It is known that the space $\mathbb{C}(E)$ of all continuous real functions on a Stonian space E (extremally disconnected compact Hausdorff space) with the usual norm is hyperconvex, and that every hyperconvex real Banach space is a space $\mathbb{C}(E)$ for some Stonian space E. Then $(\mathbf{R}^n, \|\cdot\|_\infty)$, l^∞, and L^∞ are concrete examples of hyperconvex spaces.

We follow mainly Khamsi [K].

A metric space (H, d) is said to be *hyperconvex* if for any collection of points $\{x_\alpha\}$ of H and for any collection $\{r_\alpha\}$ of nonnegative reals such that $d(x_\alpha, x_\beta) \leq r_\alpha + r_\beta$, we have

$$\bigcap_\alpha B(x_\alpha, r_\alpha) \neq \emptyset.$$

Here $B(x, r)$ denotes the closed ball with center $x \in H$ and radius $r > 0$.

For any nonempty bounded subset A of H, its *convex hull* $\operatorname{co} A$ is defined by

$$\operatorname{co} A = \bigcap \{B : B \text{ is a closed ball containing } A\}.$$

Let $\mathcal{A}(H) = \{A \subset H : A = \operatorname{co} A\}$; that is, $A \in \mathcal{A}(H)$ iff A is an intersection of closed balls. In this case we will say A is an *admissible* subset of H.

The following is known by Horvath [H2]; see also Yuan [Y]:

Lemma 7. *Any hyperconvex space (H, d) is a complete metric LC-space (H, Γ) with $\Gamma_A = \operatorname{co} A$ for each $A \in \langle H \rangle$.*

It is also known that each $X \in \mathcal{A}(H)$ itself is hyperconvex and contractible.

From Theorem 1 and Lemmas 4, 5, and 7, we have the following due to Park [P8, Corollary 4]:

Theorem 6. *Let* (H, d) *be a hyperconvex space. Then any closed compact map* $F \in \mathfrak{B}(H, H)$ *has a fixed point.*

Note that even for the subclass $\mathbb{V}(H, H) \subset \mathfrak{B}(H, H)$ Theorem 6 extends Yuan [Y, Corollary 2.5].

From Theorem 5 and Lemma 7, we have the following :

Theorem 7. *Let* (H, d) *be a hyperconvex space. Then any compact u.s.c. map* $F : H \multimap H$ *with nonempty closed* Γ*-convex values has a fixed point.*

For a single-valued map $F = f : H \to H$, Theorem 7 reduces to [P11, Theorem 7], which in turn extends a result of Espinola-Garcia; see [P11].

Since any admissible subset of a hyperconvex space is hyperconvex, we have the following :

Corollary 7.1. *Let* (H, d) *be a hyperconvex space and* $X \in \mathcal{A}(H)$*. Then any compact u.s.c. map* $F : X \multimap X$ *with nonempty closed* Γ*-convex values has a fixed point.*

The following is well-known; see Aubin and Ekeland [AE, Theorem 8]:

Lemma 7. *Let* $F, G : X \multimap Y$ *be two multimaps such that* $Fx \cap Gx \neq \emptyset$ *for all* $x \in X$*. If*

(i) F *is u.s.c. at* $x_0 \in X$*;*

(ii) Fx_0 *is compact; and*

(iii) G *is closed;*

then the multimap $F \cap G : x \mapsto Fx \cap Gx$ *is u.s.c. at* $x_0 \in X$*.*

Corollary 7.2. *Let* (H, d) *and* X *be the same as in Corollary 7.1. Let* $F : X \multimap H$ *be a compact u.s.c. multimap with nonempty closed* Γ*-convex values such that* $Fx \cap X \neq \emptyset$ *for all* $x \in X$*. Then* F *has a fixed point.*

Proof. Note that X is closed in H. Let $G : X \multimap H$ be the constant multimap defined by $Gx = X$ for all $x \in X$. Then G has closed graph $\mathrm{Gr}(G) = X \times X$ in $X \times H$. Then by Lemma 7, the map $F \cap G : X \multimap X$ is u.s.c. with nonempty closed Γ-convex values $Fx \cap X$ for $x \in X$. Moreover, it is also compact. Therefore, by Corollary 7.1, it has a fixed point $x_0 \in (F \cap G)x_0 = Fx_0 \cap X$; that is, $x_0 \in X$ and $x_0 \in Fx_0$. This completes our proof.

Since any $X \in \mathcal{A}(H)$ is Γ-convex, we immediately have the following extremely particular case of Corollary 7.2:

Corollary 7.3. *Let* H *be a hyperconvex space,* $X \in \mathcal{A}(H)$ *compact, and* $F : X \multimap \mathcal{A}(H)$ *an u.s.c. map for which* $Fx \cap X \neq \emptyset$ *for all* $x \in X$*. Then* F *has a fixed point.*

Note that Kirk and Shin [KS, Corollary 3.5] obtained the above result for a continuous map F and bounded H, and they asked whether their result remains true under the assumption that F is u.s.c. rather than continuous. We answer this question affirmatively as shown above.

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