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On Generalizations of the Meir–Keeler Type Contraction Maps: Corrections

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The original paper by three of the above authors [3] contains two mistakes. It is the purpose of this note to provide corrections for these errors. The reader should consult [3] for any special definitions or terminology not defined in this paper.

The first error involves the way in which the sequence $\{y_n\}$ is defined. In its present form, $y_{2n-1} := Sx_{2n-1} = A_{2n-1}x_{2n-2}$ and $y_{2n} := Tx_{2n} = A_{2n}x_{2n-1}$. This definition causes difficulty when one attempts to apply the definition of compatible. To remove the problem, simply select two arbitrary but fixed maps A_i and A_j, and then define $\{y_n\}$ by $y_{2n-1} := Sx_{2n-1} = A_jx_{2n-2}$ and $y_{2n} := Tx_{2n} = A_ix_{2n-1}$.

The second error occurs in the argument on pages 486 and 487, in proving that $\xi = S\xi$ and $A_i\xi = A_j\xi = \xi$. In the proof, a particular δ is chosen to satisfy condition (iv) and arrive at a contradiction. However condition (iv) does not allow such an arbitrary choice for δ. Moreover,

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Rao [2] has provided an example to show that continuity of the maps is required in order to obtain a fixed point.

There are two ways to correct this error. One is to add continuity to the hypothesis. The other is to modify the contractive definition. Adding the hypothesis of continuity would then weaken the theorem, and some of the references cited that are special cases of the theorem of the paper would not continue to be so. Instead we modify the contractive definition (iv) as follows: There exists a lower semi-continuous function $\delta: (0, \infty) \to (0, \infty)$ such that, for any $\varepsilon > 0$, $\delta(\varepsilon) > \varepsilon$, and for $x, y \in X$

$$\begin{aligned} \varepsilon &\leqslant M_{ij}(x, y) \\ &= \max\{d(Sx, Ty), d(Sx, A_i x), d(Ty, A_j y), [d(Sx, A_j y) + d(Ty, A_i x)]/2\} \\ &< \delta(\varepsilon) \end{aligned}$$

implies $d(A_i x, A_j y) < \varepsilon$.

This modification has been used by Jungck in [1].

One now replaces $486_{14} - 487^{16}$ with the following. Since (X, d) is complete, there exists a $\xi \in X$ such that $\lim y_n = \xi$. Thus $\lim Sx_{2n-1} = \lim Tx_{2n} = \lim A_i x_{2n-1} = \xi$.

Assume that S is continuous. Then $\lim SA_i x_{2n-1} = \lim SSx_{2n-1} = S\xi$.

$$\begin{aligned} d(A_i Sx_{2n-1}, S\xi) \leqslant{}& d(A_i Sx_{2n-1}, SA_i x_{2n-1}) \\ &+ d(SA_i x_{2n-1}, S\xi) \to 0 \qquad \text{as} \quad n \to \infty, \end{aligned}$$

since A_i and S are compatible. Therefore $\lim A_i Sx_{2n-1} = S\xi$.

Suppose that $\xi \neq S\xi$. Since $\lim M_n := \lim M_{ij}(Sx_{2n-1}, x_{2n}) = d(S\xi, \xi)$, let $\varepsilon = d(S\xi, \xi)$ and recall that $\delta(\varepsilon) > \varepsilon$ by definition. Since δ is lower semi-continuous, there exists an $\alpha \in (0, \varepsilon)$ such that $\delta(t) > \varepsilon$ for $t \in (\varepsilon - \alpha, \varepsilon + \alpha)$. Choose $t_0 \in (\varepsilon - \alpha, \varepsilon)$. Then $0 < t_0 < \varepsilon < \delta(t_0)$. But $\lim M_n = \varepsilon$, so there exists an n_0 such that $M_n \in (t_0, \delta(t_0))$ for $n \geqslant n_0$. Therefore by (iv), $d(A_i Sx_{2n-1}, A_j x_{2n}) \leqslant t_0 < \varepsilon$ for $n \geqslant n_0$. But $d(A_i Sx_{2n-1}, A_j x_{2n}) \to d(S\xi, \xi)$, yielding the contradiction $d(S\xi, \xi) = \varepsilon \leqslant t_0 < \varepsilon$.

Suppose now that $A_i \xi \neq \xi$. Define $\varepsilon' = d(A_i \xi, \xi)$ and let $M'_n := M_{ij}(\xi, x_{2n-1})$. Then $M'_n \to \varepsilon'$. Repeat the above argument using the lower semi-continuity of δ to obtain the contradiction $d(A_i \xi, \xi) < \varepsilon'$.

References

1. G. Jungck, Compatible mappings and common fixed points, *Internat. J. Math. Sci.* **9** (1986), 771–779.
2. K. P. R. Rao, Some coincidence points and common fixed points for self maps, *Bull. Inst. Math. Acad. Sinica* **19** (1991), 243–250.
3. B. E. Rhoades, Sehie Park, and Kwon Bae Moon, On generalizations of the Meir–Keeler type contraction maps, *J. Math. Anal. Appl.* **146** (1990), 482–494.