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ON EXTENSIONS OF THE CARISTI-KIRK FIXED POINT THEOREM

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1. Introduction

Since the appearance of the Caristi-Kirk fixed point theorem in [4], various proofs and several applications are given by a number of authors. For the literature, see Caristi [5] and Park [13]. Among those applications are fixed point theorems for maps satisfying inwardness conditions [4], results concerning normal solvability [11], metric convexity [12], characterization of metric completeness [12], [13], and many others.

Also there have appeared generalizations of the theorem. In fact, Kasahara [10] gave an L-space version of a common fixed point result for a family of the Caristi-Kirk type maps. Downing and Kirk [6] obtained a generalization and some of its applications. Also, Siegel [16] gave another generalization with simple constructive proof.

In the present paper, we show that Siegel's theorem includes the results of Downing-Kirk and Kasahara in the metric version, and we provide constructive proofs of results of Downing-Kirk and Kasahara. Also, we note that Downing-Kirk's generalization is actually equivalent to the Caristi-Kirk theorem. Simultaneously, we give a number of other equivalent formulations of the Caristi-Kirk theorem and some of their applications.

2. Siegel's Theorem

Let M and N be complete metric spaces, $f: M \to N$ be closed, that is, for $\{x_n\} \subset M$ the conditions $x_n \to x$ and $fx_n \to y$ imply $fx = y$, and $\phi : fM \to \mathbf{R}_+$ be a lower semicontinuous function.

In order to give our main result, we begin with following lemmas.

LEMMA 1. *Let* $\{x_i\}$ *be a sequence in* M *such that*

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$$\max\{d(x_i, x_{i+1}),\ d(fx_i, fx_{i+1})\} \leq \phi(fx_i) - \phi(fx_{i+1})$$

for each i, *then* $\lim_{i\to\infty} x_i = \bar{x}$ *exists and*

$$\max\{d(x_i, \bar{x}),\ d(fx_i, f\bar{x})\} \leq \phi(fx_i) - \phi(f\bar{x})$$

for each i.

Proof. Since $\{\phi(fx_i)\}$ is decreasing to some $r \geq 0$ and

$$\max\{d(x_i, x_j),\ d(fx_i, fx_j)\} \leq \phi(fx_i) - \phi(fx_j)$$

for $i \leq j$, $\{x_i\}$ and $\{fx_i\}$ are Cauchy sequences in M and N, respectively. Since M and N are complete and f is closed, there exists $\bar{x} \in M$ such that $x_i \to \bar{x}$ and $fx_i \to f\bar{x}$. On the other hand, we have

$$\begin{aligned}\max\{d(x_i, \bar{x}),\ d(fx, f\bar{x})\} &= \max\{\lim_{j\to\infty} d(x_i, x_j),\ \lim_{j\to\infty} d(fx_i, fx_j)\}\\ &\leq \phi(fx_i) - \lim_{j\to\infty}\phi(fx_j)\\ &\leq \phi(fx_i) - \phi(f\bar{x})\end{aligned}$$

from the lower semicontinuity of ϕ.

Let $h_i : M \to M$, $1 \leq i < \infty$. The countable composition of $\{h_i\}$ is defined by

$$\prod_{i=1}^{\infty} h_i(x) = \lim_{i\to\infty} h_i h_{i-1} \cdots h_1(x)$$

if the limit exists for each $x \in M$.

Let $\mathscr{H}^*$ denote the set of all $h : M \to M$ satisfying the condition

$$\max\{d(x, hx),\ d(fx, fhx)\} \leq \phi(fx) - \phi(fhx)$$

for each $x \in M$.

LEMMA 2. $\mathscr{H}^*$ *is closed under countable composition.*

Proof. For any $h_1, h_2 \in \mathscr{H}^*$,

$$\begin{aligned}&\max\{d(x, h_2h_1x),\ d(fx, fh_2h_1x)\}\\ &\quad\leq \max\{d(x, h_1x), d(fx, fh_1x)\} + \max\{d(h_1x, h_2h_1x),\ d(fh_1x, fh_2h_1x)\}\\ &\quad\leq \{\phi(fx) - \phi(fh_1x)\} + \{\phi(fh_1x) - \phi(fh_2h_1x)\} = \phi(fx) - \phi(fh_2h_1x)\end{aligned}$$

shows that $\mathscr{H}^*$ is closed under composition. By putting $x_i = h_i h_{i-1} \cdots h_1(x)$ for each $x \in X$ from Lemma 1, we have the conclusion.

For any $A \subset M$, let $r(A) = \mathrm{glb}_{x\in A}\{\phi(fx)\}$. Then $B \subset A$ implies $r(B) \geq r(A)$. For any $\mathscr{H} \subset \mathscr{H}^*$, let $\mathscr{H}(x) = \{hx \mid h \in \mathscr{H}\}$.

LEMMA 3. $\operatorname{diam} \mathscr{H}(x) \leq 2[\phi(fx) - r(\mathscr{H}(x))]$.

Proof. For any $h_1, h_2 \in \mathscr{H}$, we have

$$\begin{aligned}d(h_1x, h_2x) &\leq d(x, h_1x) + d(x, h_2x)\\ &\leq \phi(fx) - \phi(fh_1x) + \phi(fx) - \phi(fh_2x)\\ &\leq 2[\phi(fx) - r(\mathscr{H}(x))].\end{aligned}$$

The following is our version of Siegel's theorem.

THEOREM 1. *Let M and N be complete metric spaces, $f: M \to N$ be closed, $\phi: fM \to \boldsymbol{R}_+$ be a lower semicontinuous function, and $\mathcal{H}^*$ denote the family of all $h: M \to M$ satisfying*

$$\max\{(d(x, hx), d(fx, fhx)\} \leq \phi(fx) - \phi(fhx) \qquad (*)$$

for each $x \in X$. Let $\mathcal{H} \subset \mathcal{H}^*$ be closed under composition and $x_0 \in M$.

(a) *If $\mathcal{H}$ is closed under countable composition, then there exists an $\bar{h} \in \mathcal{H}$ such that $\bar{x} = \bar{h}x_0$ and $h\bar{x} = \bar{x}$ for all $h \in \mathcal{H}$.*

(b) *If each map in $\mathcal{H}$ is continuous, then there exist a sequence $\{h_i\}$ in $\mathcal{H}$ and*

$$\bar{x} = \lim_{i\to\infty} h_i h_{i-1} \cdots h_1(x_0)$$

in M such that $h\bar{x} = \bar{x}$ for each $h \in \mathcal{H}$.

Proof. Let $\{\varepsilon_i\}$ be a positive sequence converging to 0. Choose an $h_1 \in \mathcal{H}$ such that

$$\phi(fh_1x_0) - r(\mathcal{H}(x_0)) < \varepsilon_1/2.$$

Set $x_1 = h_1x_0$. Since $\mathcal{H}$ is closed under composition, $\mathcal{H}(x_1) \subset \mathcal{H}(x_0)$ and

$$\operatorname{diam} \mathcal{H}(x_1) \leq 2[\phi(fx_1) - r(\mathcal{H}(x_1))] \leq 2[\phi(fh_1x_0) - r(\mathcal{H}(x_0))] < \varepsilon_1.$$

Repeating this process, we get a sequence $\{h_i\}$ such that

$$x_{i+1} = h_i(x_i), \ \mathcal{H}(x_{i+1}) \subset \mathcal{H}(x_i) \text{ and } \operatorname{diam} \mathcal{H}(x_i) < \varepsilon_i.$$

(a) Let $\bar{h} = \prod_{i=1}^{\infty} h_i$ and $\bar{x} = \bar{h}(x_0)$. Since $\bar{x} = \prod_{j=i+1}^{\infty} h_j(x_i)$, we have $\bar{x} \in \mathcal{H}(x_i)$ for each i. Moreover, since $\lim_{i\to\infty} \operatorname{diam} \mathcal{H}(x_i) = 0$, we have $\bar{x} = \bigcap_{i=0}^{\infty} \mathcal{H}(x_i)$. Now it remains to check that $h\bar{x} = \bar{x}$ for each $h \in \mathcal{H}$. Since $h\bar{x} = h(\prod_{j=i+1}^{\infty} h_j(x_i))$, we have $h\bar{x} \in \mathcal{H}(x_i)$ for each i, whence $h\bar{x} = \bar{x}$.

(b) Let $\bar{x} = \lim_{i\to\infty} h_i h_{i-1} \cdots h_1(x_0) = \lim_{i\to\infty} x_i$. Since $\{x_j\}_{i<j} \subset \mathcal{H}(x_i)$ for each i, we have $\bar{x} \in \operatorname{cl} \mathcal{H}(x_i)$, the closure of $\mathcal{H}(x_i)$. Since $\operatorname{diam}(\operatorname{cl} \mathcal{H}(x_i)) = \operatorname{diam} \mathcal{H}(x_i)$, we have $\bar{x} = \bigcap_{i=0}^{\infty} \operatorname{cl} \mathcal{H}(x_i)$. Now observe $hx_i \in \mathcal{H}(x_i)$ for each i. Since h is continuous, for any $\varepsilon > 0$ there exists i_0 such that $B_\varepsilon(h\bar{x}) \cap \mathcal{H}(x_1) \neq \phi$, $i > i_0$. Therefore, for $i > i_0$, $d(h\bar{x}, \bar{x}) < \varepsilon + \varepsilon_i$, and since $\varepsilon_i \to 0$ we have $d(h\bar{x}, \bar{x}) \leq \varepsilon$. Since ε is arbitrary, we have $h\bar{x} = \bar{x}$.

The above proof, which is given here for the completeness, is a slight modification of that of Siegel [16]. Theorem 1 also can be deduced by the method of Brφndsted [3].

In view of Lemma 2 and Theorem 1, we have

COROLLARY. *Let M, N, f, and $\mathcal{H}$ be the same as in* Theorem 1. *Then the family $\mathcal{H}^*$ has a common fixed point. Further, if $h \in \mathcal{H}^*$ is continuous, then for any $x_0 \in M$, $\bar{x} = \lim_{i\to\infty} h^i x_0$ is a fixed point of h.*

Kasahara [10] obtained an L-space version of the first part of Corollary for the case that $M = N$ and $f = 1_M$.

By putting $N = M$ and $f = 1_M$, Theorem 1 reduces to the following

THEOREM 2 (Siegel [16]). *Let M be a complete metric space, $\phi : M \to \boldsymbol{R}_+$ be a lower semicontinuous function, and $\mathscr{H}^*$ denote the family of all $h : M \to M$ satisfying*

$$d(x, hx) \leq \phi(x) - \phi(hx)$$

for each $x \in X$. Let $\mathscr{H} \subset \mathscr{H}^*$ be closed under composition and $x_0 \in M$. Then the conclusions of Theorem 1 *hold.*

However, Theorems 1 and 2 are equivalent. To see this, in Theorem 2, let us introduce the metric

$$\rho(x, y) = \max\{d(x, y),\ d(fx, fy)\}, \qquad x, y \in M,$$

on M. Since $f : M \to N$ is closed with M and N complete, (M, ρ) is complete and $\phi \circ f$ is *l. s. c.* Hence, Theorem 2 applied to $\mathscr{H}$ on (M, ρ) yields conclusions (a) and (b) relative to (M, ρ) and, since $d(x, y) \leq \rho(x, y)$, $x, y \in M$, the same conclusions hold in (M, d).

3. Equivalent formulations

For a single map, Corollary of Theorem 1 can be stated as follows:

PROPOSITION 1. *Let M, N, f, and ϕ be the same as in* Theorem 1. *If*

(i) *a map $h : M \to M$ satisfies the condition* (*) *for each $x \in M$,*

then h has a fixed point.

PROPOSITION 2 (Downing–Kirk [6]). *Let M, N, f, and ϕ be the same as in* Theorem 1. *If*

(ii) *$h : M \to M$ is a map and c is a positive constant such that*

$$\max\{d(x, hx),\ c\,d(fx, fhx)\} \leq \phi(fx) - \phi(fhx)$$

for each $x \in M$,

then h has a fixed point.

Proof. By putting $k = \max\{1, 1/c\}$, we have

$$\max\{d(x, hx),\ d(fx, fhx)\} \leq k\,\phi(fx) - k\,\phi(fhx)$$

from (ii). Since $k\phi$ is also lower semicontinuous, Proposition 2 follows from Proposition 1.

In [6], the authors used Proposition 2 to prove surjectivity theorems for nonlinear closed maps $f : X \to Y$ where X and Y are Banach spaces.

Proposition 2 is actually an equivalent formulation of Proposition 1, for, by putting $c = 1$, (ii) implies (i).

By putting $N = M$, $f = 1_M$ (and $c = 1$) in Propositions 1 and 2, we have

PROPOSITION 3 (Caristi-Kirk [4]). *Let M be a complete metric space, $h: M \to M$ a map and $\phi: M \to \boldsymbol{R}_+$ a lower semicontinuous function. If*

$$d(x, hx) \leq \phi(x) - \phi(hx)$$

for each $x \in X$, then h has a fixed point.

Proposition 3 is useful to locate fixed points of selfmaps h such that there exist $u \in M$ and $\alpha \in [0, 1)$ satisfying

$$d(hx, h^2x) \leq \alpha\, d(x, hx)$$

for each x in $\mathrm{cl}\{h^n u\}$ and h is continuous on $\mathrm{cl}\{h^n u\}$ (Park [13]). Among such type of maps is one satisfying the condition

$$d(hy, hy) \leq \alpha \max\{d(x, y), d(x, hx), d(y, hy), [d(x, hy)+d(y, hx)]/2\}$$

for $x, y \in M$.

As for Theorems 1 and 2, note that Propositions 1 and 3 are equivalent.

We give another equivalent form of the Downing-Kirk theorem.

PROPOSITION 4. *Let M, N, f, and ϕ be the same as in* Theorem 1. *If*

(iii) *there exists a map $g: M \to N$ and a choice function $\bar{g}$ of $\{f^{-1}gx \mid x \in M\}$ such that $gM \subset fM$ and*

$$\max\{d(x, \bar{g}x), d(fx, gx)\} \leq \phi(fx) - \phi(gx)$$

for each $x \in M$,

then there exists $\bar{x} \in M$ such that $f\bar{x} = g\bar{x}$ and $\bar{g}\bar{x} = \bar{x}$.

Proof. By Propositition 1, there exists $\bar{x} \in M$ such that $\bar{g}\bar{x} = \bar{x}$. Since $\bar{x} = \bar{g}\bar{x} \in f^{-1}g\bar{x}$, we have $f\bar{x} = g\bar{x}$.

In Proposition 4, since $g = f\bar{g}$, the condition (iii) reduces to (i). Hence, Proposition 4 is equivalent to Proposition 1.

We can prove Proposition 4 by the method of the proof of the main result of [6], which is based on the idea of Brøndsted [2].

The function $\phi: M \to \boldsymbol{R}_+$ in the Caristi-Kirk theorem and in other results in this paper can be replaced by $\phi: M \to \boldsymbol{R}$ bounded from below. However, the condition "bounded from below" can not be dispensable. For example, if we put $M = N = \boldsymbol{R}$, $f = 1_R$, $\bar{g}x = x - 1$, $\phi = 1_R$ in Proposition 4, then $\bar{g}$ has no fixed point. Also the condition $gM \subset fM$ in Proposition 4 can not be dispensable. For example, if we put $M = N = \boldsymbol{R}_+$, $fx = x+1$, $g = 1_{R_+}$, $\phi = 1_{R_+}$ in Proposition 4, then f and g have no coincidence.

By putting $N = M$ and $f = 1_M$, Proposition 4 reduces to the Caristi-Kirk theorem. Further, by putting $N = M$ and $g = 1_M$, Proposition 4 reduces to

COROLLARY 1. *Let M be a complete metric space, $f: M \to M$ a closed surjection, and $\phi: M \to \boldsymbol{R}_+$ a lower semicontinuous function. If for any $x \in X$*

there exists $y \in f^{-1}x$ *such that*

$$\max\{d(x, y),\ d(x, fx)\} \leq \phi(fx) - \phi(x),$$

then f *has a fixed point.*

From Corollary 1, we have

COROLLARY 2. *Let* M *be a complete metric space,* $f : M \to M$ *a closed surjection, and* $\phi : M \to \boldsymbol{R}_+$ *a lower semicontinuous function. If for any* $y \in M$,

$$\max\{d(y, fy),\ d(fy, f^2y)\} \leq \phi(f^2y) - \phi(fy)$$

holds, then f *has a fixed point.*

The following is also equivalent to Proposition 1.

PROPOSITION 5. *Let* M *and* N *be complete metric spaces,* $f : M \to N$ *a closed map, and* $\phi : fM \to \boldsymbol{R}$ *a lower semicontinuous function bounded from below. Then there exists a point* $p \in X$ *such that*

$$\phi(fp) - \phi(fx) < \max\{d(p, x),\ d(fp, fx)\}$$

for each $x \in M$ *other than* p.

This was given in [15]. From Proposition 5, we obtain

PROPOSITION 6 (Ekeland [7], [8]). *Every lower semicontinuous function* ϕ *from a complete metric space* M *into* $\boldsymbol{R}_+$ *has a* d*-point* p *in* M*, that is, we have*

$$\phi(p) - \phi(x) < d(p, x)$$

for every other point x *in* M.

Proposition 6 is equivalent to the Caristi–Kirk Theorem (see [1]).

In view of Proposition 6, we give another equivalent formulations of the Caristi–Kirk theorem.

PROPOSITION 7. *Let* X *be a set,* M *a complete metric space, and* $f,\ g : X \to M$ *maps such that*

(1) f *is surjective, and*

(2) *there exists a lower semicontinuous function* $\phi : M \to \boldsymbol{R}_+$ *satisfying*

$$d(fx, gx) \leq \phi(fx) - \phi(gx)$$

for each $x \in X$.

Then f *and* g *have a coincidence.*

Proof. By Proposition 6, ϕ has a d-point $p \in M$. Let $x \in f^{-1}p$. Suppose $fx \neq gx$. Since $p = fx$ and $gx \in M$ we have

$$\phi(fx) - \phi(gx) < d(fx, gx),$$

which contradicts (2).

By putting $X = M$ and $f = 1_M$, Proposition 7 reduces to the Caristi-Kirk theorem.

The condition (2) can be replaced by various contractive type conditions without affecting the conclusion. In fact, Goebel [9] used the condition

(2)′ $d(gx, gy) \leq \alpha d(fx, fy)$, $\alpha \in [0, 1)$,

and gave an application to the unique existence of solution of differential equation of the form $x' = f(t, x)$. Park [14] extended this fact by using the Meir-Keeler type contractive condition:

(2)″ for a given $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ such that for $x, y \in X$,

$$\varepsilon \leq d(fx, fy) < \varepsilon + \delta \text{ implies } d(gx, gy) < \varepsilon,$$

and $fx = fy$ implies $gx = gy$.

From Proposition 7, we obtain a Downing-Kirk type result as follows:

PROPOSITION 8. *Let M and N be metric spaces and $h : M \to M$ a map. Suppose there exist a map $f : M \to N$, a lower semicontinuous function $\phi : fM \to \mathbf{R}_+$, and a constant $c > 0$, such that fM is complete and for each $x \in M$,*

$$\max\{d(x, hx), c\, d(fx, fhx)\} \leq \phi(fx) - \phi(fhx).$$

Then h has a fixed point.

Proof. Since $(1/c)\phi$ is lower semicontinuous, putting $fh = g$ and $(1/c)\phi = \phi'$, we have

$$d(fx, gx) \leq \phi'(fx) - \phi'(gx).$$

Therefore, by Proposition 7, f and fh have a coincidence $\bar{x} \in X$. Since

$$d(\bar{x}, h\bar{x}) \leq \phi(f\bar{x}) - \phi(fh\bar{x}),$$

we have $\bar{x} = h\bar{x}$.

Note that, by putting $X = M$, $f = 1_M$, and $c = 1$, Proposition 8 reduces to Proposition 3.

Moreover, by putting $X = M$ and $g = 1_M$, Proposition 8 reduces to

PROPOSITION 9. *Let M be a complete metric space and $f : M \to M$ be a surjection. If there exists a lower semicontinuous function $\phi : M \to \mathbf{R}_+$ satisfying*

$$d(x, fx) \leq \phi(fx) - \phi(x)$$

for $x \in M$, then f has a fixed point.

Proposition 9 may be used to show the existence of fixed point of certain maps f satisfying

$$d(x, fx) \leq \alpha\, d(fx, f^2x), \quad \alpha \in [0, 1)$$

for each $x \in M$, since we can put

$$\phi(x)=\frac{\alpha}{1-\alpha}d(x,\ fx).$$

Among such type are maps f satisfying

$$d(x,\ y)\leq\alpha\max\{d(fx,\ fy),\ d(x,\ fx),\ d(y,\ fy),\ [d(x,\ fy)+d(y,\ fx)]/2\}$$

for $x,\ y\in M$.

Note also that in Proposition 9, choosing $y\in f^{-1}x$, we have

$$d(y,\ fy)\leq\phi(y)-\phi(fy)$$

for each $y\in M$. This shows that the Caristi-Kirk theorem follows from Proposition 9.

Finally, from Proposition 7 we have

COROLLARY. *Let F be a selfmap of a Banach space B, α, β numbers, $|\alpha|\neq|\beta|$, and $F_{\alpha,\beta}=\alpha 1_B+\beta F$. If $F_{\alpha,\beta}(B)\subset F_{\beta,\alpha}(B)$, $F_{\beta,\alpha}(B)$ is closed in B, and if there exists a lower semicontinuous function $\phi:B\rightarrow\mathbf{R}_+$ satisfying*

$$\|F_{\alpha,\beta}(x)-F_{\beta,\alpha}(x)\|\leq\phi(F_{\beta,\alpha}(x))-\phi(F_{\alpha,\beta}(x))$$

for any $x\in B$, then F has exactly one fixed point.

Proof. From Proposition 7 $F_{\alpha,\beta}$ and $F_{\beta,\alpha}$ have a coincidence $x\in B$. It is clear that x is the unique fixed point of F.

Note that Goebel [9] showed the same result using the condition

$$\|F_{\alpha,\beta}(x)-F_{\alpha,\beta}(y)\|\leq k\,\|F_{\beta,\alpha}(x)-F_{\beta,\alpha}(y)\|,\quad 0\leq k<1,$$

for $x,\ y\in B$, instead of the inequality in the above Corollary. In [14], this result was extended to the Meir-Keeler type condition similar to $(2)''$.

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